

Baryon resonances in a chiral confining model.

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In this two part series a chiral confining model of baryons is used to describe low-lying negative parity resonances N^* , Δ^* , Λ^* and Σ^* in the mean field approximation. A physical baryon in this model consists of interacting valence quarks, mesons and a color and chiral singlet hybrid field coexisting inside a dynamically generated confining region. This first paper presents the quark contribution to the masses and wave functions of negative parity baryons calculated with an effective spin-isospin dependent instanton induced interaction. It does not include meson exchanges between quarks. The three-quark wave functions are used to calculate meson-excited baryon vertex functions to lowest order in meson-quark coupling. When the baryons are on mass-shell each of these vertex functions is a product of a coupling constant and a form factor. As examples, quark contributions to N^* hadronic form factors as well as axial coupling constants are extracted from the vertex functions and problems with the analytical behaviour of the model form factors are discussed. The second paper will examine the mesonic corrections to excited baryon properties in the heavy baryon and one-loop approximations.

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I. INTRODUCTION

An interesting and sensitive test for models of baryon structure will soon be performed at the recently completed Jefferson Lab facility when the radiative decay widths of the first two excited states of the Λ hyperon, $\Lambda(1405)$ and $\Lambda(1520)$, will be determined directly in a quadrupole coincidence measurement experiment [1]. There are four such widths to be measured, two for each excited hyperon radiatively decaying into ground states Λ^0 and Σ^0 , and so far none of the predictions of various models of baryons inspired by Quantum Chromodynamics (QCD) agree with each other [2,3,4,5]. Calculation of these widths is extremely model dependent involving the wave functions of excited hyperons as well as transition operators both of which must be determined in a self consistent manner. It is therefore a challenge for *any* model of baryon structure to predict correctly all four widths. Discrepancies among existing model predictions are large enough to be able to distinguish experimentally among various results. But unfortunately, presently available experimental data on the four radiative decay widths either disagree by an order of magnitude [6,7] or have been extracted in a model dependent manner [8].

Phenomenologically this test is particularly welcome since almost all the models of baryon structure can describe the static properties of the ground state octet and decuplet baryons rather well, and therefore it is very difficult to judge the success of any given model based on its description of the ground states alone. In addition, it is often the case that a given model which can reproduce the properties of ground state baryons will run into difficulties when extended to describe the excited states. An example of this difficulty is the removal of the center-of-mass motion in relativistic quark models where no rigorous method to project out the spurious degrees of freedom exist. Thus a successful description of ground state baryons in any model is a necessary but not a sufficient condition for the model to be realistic, and the upcoming direct precision measurements of the excited hyperon radiative decay widths at Jefferson Lab present an excellent testing ground for any model of baryons under development.

One such model is the chiral confining model (CCM) of baryons [9,10] where a baryon is described as a color singlet composite object consisting of interacting valence quarks, mesons and a color and chiral singlet hybrid field χ . The model produces absolute quark confinement using a mechanism introduced by Nielsen and Pätikos [11] to dynamically generate a confining region, or a “bag”, inside which the above constituents coexist to form a baryon. Although the model has not been directly derived from QCD, it is at least consistent with the essential properties of the underlying theory of strong interactions such as chiral invariance and the behaviour in the limit of large number of colors, N_c .

The initial version of the CCM [12] revealed two major problems. The model prediction for the product of the nucleon mass and the quark rms radius was too large, while that for $N - \Delta$ mass splitting was too small. McGovern, *et al* [13] showed there was an approximate scaling law for the CCM which stated that if either the mass or the size of the nucleon was kept fixed at some particular value all other predictions of the CCM were also more or less fixed. The problems were resolved [14] by extending the model to include the 't Hooft interaction [15,16]. By fitting the $N - \Delta$ mass difference to determine the strength of the effective 't Hooft interaction and using simple estimates to correct for the center-of-mass motion, the prediction for the product of the nucleon mass and the quark rms radius were reduced to an acceptable value in the range of 3.8 to 4.4, the empirical value being about 3.5.

In this two-part series the CCM is extended to the strangeness sector and is used to examine the properties of low-lying negative parity N^* , Δ^* , Λ^* and Σ^* baryons in the mean field approximation. Excited baryons are regarded as RPA excitations of the ground state baryons and includes both particle-hole excitations of the valence quarks as well as creation of pseudoscalar mesons. The first paper considers excited baryons built with valence quarks in $S_{\frac{1}{2}}$, $P_{\frac{1}{2}}$ and $P_{\frac{3}{2}}$ states and examines the contribution of such states to the masses, wave functions and hadronic form factors using a spin-isospin dependent instanton induced interaction [15,16]. Baryons obtained with only the valence quarks are referred to as “bare” baryons. Mesonic corrections to properties of bare excited baryon will be presented in a following paper.

The role of gluons in the CCM appears in three distinct ways:

First, the gluon condensate of the vacuum manifests itself through the appearance of the color dielectric function [11]. The fluctuation of the gluon condensate appears in the form a 0^{++} hybrid field [9].

Second, the instanton, a feature of *classical* Euclidean QCD, appears through the $2N_f$ (N_f being the number of flavors) fermion interaction introduced by 't Hooft [15,16] to represent approximately the role of instantons on the quark contribution to the partition function. In the absence of this idea it would not be possible to include the role of instantons in a mean field theory in the Minkowski world.

Third, the role of the exchange of *quantum* gluons is described through meson exchanges [17,18,19].

Note that perturbative one-gluon exchange (OGE) interaction cannot be included in this model as that would lead to double counting. Since OGE interaction is widely used in quark models by many authors, the differences between three-quark wave functions obtained with the instanton induced and OGE interactions are examined.

In order to diagonalize the bare baryon Hamiltonian, H_{Bare} , it is necessary to remove the spurious components in the excited baryon wave functions corresponding to the translation of the center-of-mass of the composite baryon. For the CCM, where a baryon is a composite object consisting of relativistic quarks, mesons and the χ field, seeking an exact solution to this problem is an impractical task. Thus in this work a prescription originally used in the study of excited baryons in the MIT bag model by DeGrand [20] is used to approximately eliminate the unphysical states. The method is able to predict the observed number of low-lying negative parity N^* and Δ^* states but is strictly valid only in the non-relativistic and $SU(6)$ spin-flavor limits. A simple test of this prescription is presented in the Appendix B in order to estimate the relative amount of spurious component present in the excited hyperon wave function for different strange quark masses. After eliminating the spurious states H_{Bare} is diagonalized by assuming equal masses for up, down and strange quarks to be consistent with the center-of-mass removal prescription. Corrections from flavor symmetry breaking to the hyperon masses are given to first order by modifying the single particle energies of strange quarks after diagonalization.

The three-quark wave functions for excited baryons are used to calculate meson-bare excited baryon vertex functions to lowest order in meson-quark coupling. When the baryons are on mass-shell each of these vertex functions is a product of a coupling constant and a form factor. Meson-bare baryon form factors are extracted from the vertex functions and used to determine the quark contribution to the axial coupling constants of negative parity nucleons. It is pointed out that these hadronic form factors do not have the correct analytical behaviour in the complex q -plane. In the second paper these vertex functions are used to determine meson cloud contribution to physical baryon properties in the heavy baryon and one-loop approximations.

Essential features of the CCM are presented in the following section together with single particle energies and spinors of confined quarks in the ground and relevant excited states. The model Hamiltonian for bare baryons is then defined by using the 't Hooft interaction as the residual interaction between quarks. In Section III, this Hamiltonian is diagonalized and bare masses and wave functions of negative parity baryons N^* , Δ^* , Λ^* and Σ^* are presented and compared to other model calculations. These three-quark wave functions are used in Section IV to calculate meson-bare excited baryon vertex functions. Corresponding form factors for negative parity N^* resonances and their axial coupling constants are also presented as examples and problems with the results discussed. Finally, the paper is summarized in Section V.

II. THE CHIRAL CONFINING MODEL AND THE 'T HOOFT INTERACTION

A. The Chiral Confining Model of Baryons

The CCM [9,10] is based on the color dielectric theory proposed by Nielsen and Pàtkos [11]. These authors introduced two collective variables, $K(x)$, a color singlet, charge-conjugation even object, and $B_\mu^a(x)$, a coarse-grained gluon field expressed in terms of averages of link operators for loops contained in a hypercube of side L . Upon integrating out the QCD gluon fields in favor of these new collective variables Nielsen and Pàtkos obtained the following Lagrangian in the form of a derivative expansion,

$$\mathcal{L}_{NP} = \bar{\psi}(x) \left[iK(x) \frac{1}{2} \overleftrightarrow{\not{D}} - K(x)m_q - g\not{B}(x) \right] \psi(x) - \frac{K(x)^4}{4} G_{\mu\nu}^a G^{a\mu\nu} + \dots \quad (2.1)$$

In terms of the gauge field, B_μ^a/K , the coarse grained field tensor is

$$G_{\mu\nu}^a = \partial_\mu \frac{B_\nu^a}{K} - \partial_\nu \frac{B_\mu^a}{K} + f^{abc} \frac{B_\mu^b}{K} \frac{B_\nu^c}{K}. \quad (2.2)$$

From the gluonic term one identifies $\epsilon = K^4$ as the color dielectric function. Nielsen and Pàtkos conjectured that

$$\langle K \rangle_{vac} = 0. \quad (2.3)$$

The CCM adds four new features to the Nielsen and Pàtkos model. First, the conjecture shown in Eq. (2.3), crucial for the CCM, was justified from the lattice gauge point of view by Lee *et al.* [21]. It is customary to introduce the field $\chi(x)$ as

$$K(x) = g_\chi \chi(x), \quad (2.4)$$

whereupon g_χ has the dimensions of L . The second feature in CCM is the demonstration, through large N_c analysis [9], that $\chi(x)$ is a hybrid and not a glueball field¹. This is crucial for proper large N_c behavior of CCM itself.

The third feature in CCM is the conjecture that there exists a suitable gauge in which the coarse-grained gluon field $B_\mu^a(x)$ may be integrated out in favor of meson fields. The meson fields were described with the Gell-Mann L'evy linear σ -model [22]. This allowed the introduction, through $\langle 0 | \sigma | 0 \rangle = -93 \text{ MeV}$, of an ever-present mass like term for the quarks in a perfectly chiral invariant manner. Thus field variables in the CCM are quarks, mesons and the hybrid field χ . The vanishing of $K(x)$ in the vacuum means that

$$\langle 0 | \chi | 0 \rangle = 0. \quad (2.5)$$

The meson-quark coupling may involve a factor K^n . The fourth feature of CCM is the fixing of the exponent n . This was found by comparing the K -dependence of the four-fermion interaction obtained by integrating out the B_μ^a fields of the Nielsen and Pàtkos Lagrangian, Eq. (2.1), with that obtained by integrating out the mesons in the CCM Lagrangian. It was found that $n = -1$.

Finally, canonical quark fields were introduced using the transformation $\sqrt{g_\chi \chi} \psi \rightarrow \psi$. In terms of the canonical quark fields the effective CCM Lagrangian, extended to SU(3) flavor group is

$$\mathcal{L}_{\text{CCM}} = \bar{\psi} \left[\frac{i}{2} \overleftrightarrow{\not{\partial}} - m_q + \frac{g_\phi}{(g_\chi \chi)^2} \left(\sigma + i\gamma_5 \lambda^a \phi^a + \dots \right) \right] \psi + \mathcal{L}_\chi + \mathcal{L}_{\text{meson}}. \quad (2.6)$$

where ϕ^a are the pseudoscalar octet mesons with flavor index a , ($a = 1 \rightarrow 8$), and the ellipses refer to other mesons such as ω, ρ or A . The Gell-Mann matrices λ^a are normalized to $\text{Tr}(\lambda^a \lambda^b) = 2\delta_{ab}$. The inclusion of vector mesons requires that CCM Lagrangian use the Lee and Nieh's form for $\mathcal{L}_{\text{meson}}$ [23].

Two simple forms for the χ potential, pure mass and quartic, have been considered. This paper will use only the pure mass form given by

$$\mathcal{L}_\chi = \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \frac{1}{2} m_\chi^2 \chi^2. \quad (2.7)$$

Extensive SU(2) calculations have shown that the meson fields produced by the valence quarks themselves play relatively less important role in the baryon structure. Hence a reasonable approximation is to set the mesons fields at their vacuum values. Thus

$$\langle 0 | \sigma | 0 \rangle = -F_\pi, \quad (2.8)$$

and all other meson fields are fixed at the value zero. In addition, the parameter g_ϕ in Eq. (2.6) is set to g_π . The resulting model is called the Toy model. It must be understood that it is used solely to obtain reasonable approximations to the valence spinors and the χ field. For example, in order to calculate meson-baryon coupling constants the full CCM Lagrangian must be used to obtain the meson source currents.

The Toy model Lagrangian is

$$\mathcal{L}_{\text{Toy}} = \bar{\psi} \left(\frac{i}{2} \overleftrightarrow{\not{\partial}} - m_q - \frac{g_\pi F_\pi}{(g_\chi \chi)^2} \right) \psi + \mathcal{L}_\chi. \quad (2.9)$$

The bag formation mechanism depends on two critical ingredients specified by Eqs. (2.5), $\langle 0 | \chi | 0 \rangle = 0$, and (2.8), $\langle 0 | \sigma | 0 \rangle \neq 0$ and incorporated in the Toy model. The Euler Lagrange equation for the quark field from the Toy model is

$$(i\not{\partial} - m_q)\psi = -\frac{g_\pi F_\pi}{(g_\chi \chi)^2} \psi. \quad (2.10)$$

Thus a quark cannot exist in the region where $\chi = 0$. However because of the coupling between quarks and the χ field, valence quarks change the value of the χ field away from its vacuum value and make it nonzero. This region is the dynamically created bag where a quark can exist.

¹While the properties of the QCD vacuum is dominated by the glueball field, changes produced by baryon matter are dominated by hybrids.

There are four adjustable parameters in \mathcal{L}_{Toy} , m_q , m_χ , g_χ and g_π . The first is left fixed at the SU(3) flavor limit value of $m_q = 7.5$ MeV. Corrections to hyperon masses from SU(3) breaking are discussed below. McGovern *et al* [13] showed that the Toy model possesses approximate scaling and, as a consequence, the combination $[g_\pi F_\pi m_\chi^2 / g_\chi^2]^{\frac{1}{3}}$, which has the dimensions of L^{-1} , is the only operative parameter, not the individual values of m_χ , g_χ and g_π . In the description of ground state properties of N and Δ [14,24] these parameters are fixed to be $m_\chi = 1200$ MeV, $g_\chi = 0.08$ MeV $^{-1}$ and $g_\pi = 4$. The 't Hooft interaction introduces two additional parameters, the interaction strength C_s and the regulator parameter r_c , which are also fixed in [14,24] to be $C_s = 1.6$ fm 2 and $r_c = 0.25$ fm. These values of model parameters are used in the present work.

B. The toy model

The toy CCM Hamiltonian corresponding to \mathcal{L}_{Toy} of Eq. (2.9) is given by

$$H_{\text{Toy}} = \int d^3r \left[\psi^\dagger(\vec{r}) \left\{ -i\vec{\alpha} \cdot \vec{\nabla} + \beta \left(m_q + \frac{g_\pi F_\pi}{(g_\chi \chi)^2} \right) \right\} \psi(\vec{r}) + \frac{1}{2} [(\vec{\nabla} \chi)^2 + m_\chi^2 \chi^2] \right], \quad (2.11)$$

and the corresponding energy function when the three valence quarks occupy the same spinor state $u(\vec{r})$ is

$$E_{\text{Toy}} = \int d^3r u^\dagger(\vec{r}) \left\{ -i\vec{\alpha} \cdot \vec{\nabla} + \beta \left(m_q + \frac{g_\pi F_\pi}{(g_\chi \chi)^2} \right) \right\} u(\vec{r}) + E_\chi, \quad (2.12)$$

where E_χ is the *classical* χ field energy given by

$$E_\chi = \int d^3r \frac{1}{2} [(\vec{\nabla} \chi)^2 + m_\chi^2 \chi^2]. \quad (2.13)$$

In the mean field treatment of the CCM the ground state problem is solved by making E_{Toy} of Eq. (2.12) stationary with respect to the quark spinor $u(\vec{r})$ and the χ field, subject to the normalization $\int d^3r u^\dagger(\vec{r}) u(\vec{r}) = 1$.

Let $\chi^{(0)}$ be the χ field resulting from the mean field treatment of the ground state and $E_\chi^{(0)}$ the corresponding classical χ field energy. Then from Eq. (2.11), the CCM Hamiltonian describing non-interacting quarks moving in a fixed mean χ field is

$$H_{\text{Toy}}^{(0)} = \int d^3r \psi^\dagger(\vec{r}) \left\{ -i\vec{\alpha} \cdot \vec{\nabla} + \beta \left(m_q + \frac{g_\pi F_\pi}{(g_\chi \chi^{(0)})^2} \right) \right\} \psi(\vec{r}) + E_\chi^{(0)}. \quad (2.14)$$

$H_{\text{Toy}}^{(0)}$ describes the foundation of the quark model used in this work. The lowest energy eigenspinor of this Hamiltonian is the one which makes the energy function given by Eq. (2.12) stationary. The numerical value of $E_\chi^{(0)}$ is found to be about 300 MeV [24]. Being the same for all baryon states it plays no role in determining the spin-flavor compositions of excited baryons. In the present quark shell model description of excited baryons, the low-lying negative parity states are constructed by exciting a single quark from the ground $S_{1/2}$ state to either the $P_{1/2}$ or $P_{3/2}$ state. Adapting the notation $S \equiv S_{1/2}$, $P \equiv P_{1/2}$ and $A \equiv P_{3/2}$, quark spinors for S , P and A states are defined as

$$u_S(\vec{r}) \equiv \begin{pmatrix} G_0(r) \\ i\vec{\sigma} \cdot \hat{r} F_0(r) \end{pmatrix} \xi_S(\hat{r}), \quad (2.15)$$

$$u_P(\vec{r}) \equiv \begin{pmatrix} G_2(r) \\ i\vec{\sigma} \cdot \hat{r} F_2(r) \end{pmatrix} \xi_P(\hat{r}), \quad (2.16)$$

$$u_A(\vec{r}) \equiv \begin{pmatrix} G_3(r) \\ i\vec{\sigma} \cdot \hat{r} F_3(r) \end{pmatrix} \xi_A(\hat{r}). \quad (2.17)$$

These spinors satisfy the eigenvalue equation

$$H_{\text{Toy}}^{(0)} u_i(\vec{r}) = \epsilon_i u_i(\vec{r}), \quad (2.18)$$

where $i = S, P$ or A .

Figure 1 shows the upper, $G_i(r)$, and the lower, $F_i(r)$, radial functions in the quark spinor for S , P and A states defined in Eqs. (2.15) to (2.17) for $m_u = m_d = 7.5$ MeV and $m_s = 300$ MeV. Also shown in Figure 1a is the radial

profile for the $\chi^{(0)}$ field which appears in $H_{\text{Toy}}^{(0)}$. The eigenvalues of S , P and A spinors of $H_{\text{Toy}}^{(0)}$ as defined in Eq. (2.18) for $m_q = m_{u,d}$ and $m_q = m_s$ are presented in Table I. In addition, Table II shows the values of the quark scalar charge \mathcal{S}_i defined as,

$$\mathcal{S}_i \equiv \int d^3r \bar{u}_i u_i = \int dr r^2 (G^2 - F^2)_i, \quad (2.19)$$

with the normalization

$$\int d^3r u_i^\dagger u_j = \delta_{ij}. \quad (2.20)$$

As expected the higher the current quark mass m_q smaller is the lower components of the quark spinors and higher are the scalar charges. Note, however, that changes in \mathcal{S}_i are 40% and larger.

In the present work the effect of quark mass difference $m_s - m_{u,d}$ is treated perturbatively. Only the spinors with $m_u = m_d = m_s = 7.5$ MeV are used to evaluate matrix elements. SU(3) breaking effects on hyperon masses are estimated using the quark scalar charge given in Eq. (2.19) as follows. Three-quark wave functions for excited hyperons are first obtained by diagonalizing the mass matrix including the 't Hooft interaction. In general these wave functions are linear combinations of states with the excited quark in either the P or the A state. After diagonalization, single particle energies for strange quarks are modified from ϵ_i to $\epsilon_i + \delta\epsilon_i$ where

$$\delta\epsilon_i = (m_s - m_q) \int d^3r \bar{u}_i u_i. \quad (2.21)$$

Using the scalar charges given in Table II for $m_q = 7.5$ MeV, the numerical values for $\delta\epsilon_i$ are

$$\delta\epsilon_S = (m_s - m_q) \int d^3r \bar{u}_S u_S = 184 \text{ MeV}, \quad (2.22)$$

$$\delta\epsilon_P = (m_s - m_q) \int d^3r \bar{u}_P u_P = 100 \text{ MeV}, \quad (2.23)$$

$$\delta\epsilon_A = (m_s - m_q) \int d^3r \bar{u}_A u_A = 163 \text{ MeV}. \quad (2.24)$$

For example, consider a state where the strange quark is excited to the A state while the remaining two non-strange quarks are in their ground states. The sum of single quark energies for this state is modified from $2\epsilon_S + \epsilon_A = 1170$ MeV to $2\epsilon_S + \epsilon_A + \delta\epsilon_A = 1333$ MeV where ϵ_i 's are the single quark energy eigenvalues obtained with $m_q = 7.5$ MeV.

C. The 't Hooft interaction

Phenomenological consequences of the 't Hooft interaction in baryon structure have been investigated using various quark models by Kochelev [25], Shuryak and Rosner [26], Oka and Takeuchi [27], Blask *et al.* [28], Kim and Banerjee [14,24], Klabučar [29] and most recently by Takeuchi [30,31]. The role of the 't Hooft interaction in the excited baryon mass spectra has previously been examined by Blask *et al.* [28] and Takeuchi [30,31] using variants of the non-relativistic quark model. The model used by Blask *et al.* consists of a linear confining potential simulating a string-like confinement and the 't Hooft interaction which was used as the residual interaction without any OGE interactions. Using this model they managed to reproduce the qualitative features of meson and baryon mass spectra up to masses of order 2 GeV including the $\eta - \eta'$ splitting. Shuryak and Rosner [26] also used a similar constituent quark model to parametrize the ground state baryon mass spectrum using only the two-body effective instanton induced force. Thus the constituent quark model seems to give a reasonable description of the low-lying baryon and meson mass spectra with either the OGE or the 't Hooft interaction as the residual interaction between a pair of quarks.

The Hamiltonian for the constituent quark model used by Oka and Takeuchi [27] and later by Takeuchi [30,31] includes both the OGE and 't Hooft interactions in the following way,

$$H = K + V_{\text{conf}} + (1 - p_{\text{III}})V_{\text{OGE}} + p_{\text{III}}V_{\text{'t Hooft}}, \quad (2.25)$$

where K and V_{conf} are quark kinetic energy and confining potentials, respectively. The parameter p_{III} is introduced to adjust the strength of the OGE potential, V_{OGE} , relative to that of the 't Hooft interaction potential, $V_{\text{'t Hooft}}$.² This parameter is in the range of 0.3 to 0.4 when the model is used to fit the $\eta - \eta'$ mass splitting. Takeuchi used this Hamiltonian to calculate the masses of excited nucleons up to the $N = 2$ harmonic oscillator level and the nucleon–nucleon force in the quark cluster model [30,31].

It was realized early in the development of the NRQM by Isgur and Karl [32,33] that the low-lying negative and positive parity non-strange baryon mass spectra can be well reproduced if the spin–orbit force from the OGE interaction is somehow suppressed. In calculating the excited nucleon masses Takeuchi found that the spin–orbit force from the 't Hooft interaction mostly cancels the spin–orbit force originating in the OGE interaction. However, this cancellation does not occur in the nucleon–nucleon force where the combination of the OGE and 't Hooft interactions produces a strong spin–orbit component. In the same work Takeuchi also examined the low-lying negative parity N^* and Δ^* mass spectra using the MIT bag model with both OGE and 't Hooft interactions and found that the cancellation seem to persist for bag models as well [30]. Unfortunately, however, both Blask *et al.* and Takeuchi focused only on the masses of the excited states and did not investigate the spin–flavor compositions of the baryon wave functions. In this work close attention is paid to the spin–flavor contents of excited baryons since they determine the meson–baryon vertex functions as well as radiative transition amplitudes to the ground states.

Kim and Banerjee developed a program to incorporate the 't Hooft interaction into the CCM [14] and their method is used in this work. They pointed out that although the strength of the 't Hooft interaction in QCD can be estimated using Shuryak's instanton liquid model of the QCD vacuum [34], the result cannot be used directly in the CCM where the block–spinning procedure of Nielsen and Pätikos [11] modifies the effective 't Hooft interaction Lagrangian. Because of this reason the strength parameter C_s was fixed by fitting the $N - \Delta$ mass splitting including both the 't Hooft and perturbative one–pion exchange (OPE) interactions. The results for the mass and the size of the nucleon showed remarkable improvement relative to earlier CCM results as mentioned in the Introduction.

However, the neutron charge radius turned out to be too small. Isgur *et al.* [35,36] had pointed out that non-perturbative effects of OGE can explain the size of $\langle r^2 \rangle_{\text{neutron}}$. Similar effects exist in the CCM when the 't Hooft interaction is used in conjunction with perturbative OPE interaction as the source of spin and isospin dependent forces.³ The nucleon problem was redone in the CCM in [24] by mixing all seven possible configurations made out of $1S_{\frac{1}{2}}$, $1P_{\frac{1}{2}}$ and $1P_{\frac{3}{2}}$ single quark states. In addition, two configurations of the type $(1S_{\frac{1}{2}})^2 2S_{\frac{1}{2}}$ and one of the type $(1S_{\frac{1}{2}})^2 1D_{\frac{3}{2}}$ have also been used. The calculation yielded a very good fit for $\langle r^2 \rangle_{\text{neutron}}$.⁴

For N_f flavors the 't Hooft interaction consists of a sum of one, two and up to N_f –body terms [16]. For 3 flavors the interaction is given by

$$\begin{aligned} \mathcal{L}'_{\text{'t Hooft}} = & -am_1m_2m_3 \sum_i \bar{\psi}_i R \psi_{iL}/m_i + b \left(m_3 \bar{\psi}_1 R \psi_{1L} \bar{\psi}_2 R \psi_{2L} + \text{permut.} \right) \\ & + c \bar{\psi}_1 R \psi_{1L} \bar{\psi}_2 R \psi_{2L} \bar{\psi}_3 R \psi_{3L} \\ & + \left[\left(Am_3 - A' \bar{\psi}_3 R \psi_{3L} \right) \bar{\psi}_1 R \lambda^a \psi_{1L} \bar{\psi}_2 R \lambda^a \psi_{2L} \right] \\ & + Bd^{abc} \left[\bar{\psi}_1 R \sigma_{\mu\nu} \lambda^a \psi_{1L} \bar{\psi}_2 R \sigma_{\mu\nu} \lambda^b \psi_{2L} \bar{\psi}_3 R \lambda^c \psi_{3L} + \text{permut.} \right] \\ & + Cd^{abc} \bar{\psi}_1 R \lambda^a \psi_{1L} \bar{\psi}_2 R \lambda^b \psi_{2L} \bar{\psi}_3 R \lambda^c \psi_{3L} \\ & + Df^{abc} \bar{\psi}_1 R \sigma_{\mu\nu} \lambda^a \psi_{1L} \bar{\psi}_2 R \sigma_{\nu\gamma} \lambda^b \psi_{2L} \bar{\psi}_3 R \sigma_{\gamma\mu} \lambda^c \psi_{3L}. \end{aligned} \quad (2.26)$$

where $i = 1, 2, 3$ refer to the three flavors and $\psi_{iR/L}$ is the right/left-handed quark spinor with flavor i and current quark mass m_i . The constants, a, b, c, A, A', B, C and D depend upon the distribution of instanton size in the instanton liquid vacuum. When applied to the CCM the one–body term induces a change in the current quark mass of

²This Hamiltonian treats the classical 't Hooft and quantum OGE interactions on an equal footing. In the CCM gluon exchanges are effectively taken into account by meson exchanges which in the mean field theory are included as a quantum corrections to the classical 't Hooft interaction. Thus, assuming a Hamiltonian of this form would lead to overcounting in the present work.

³Recall that in the CCM contributions from gluon exchanges, including the OGE, is effectively included in the OPE interaction

⁴Furthermore, with the pure mass χ potential used in this work, the coupling constant $g_{\pi NN^*}$ was largest for the lowest N^* state and sharply smaller for the remaining ones.

about 7%. In practice, this small modification is ignored and Kim and Banerjee retained only the dominant two-body interaction for the description of the $N - \Delta$ system [14]. The same approach was used by Kochelev [25] and Takeuchi [30], all of whom used bag models to calculate the effects of the 't Hooft interaction on baryon masses.

Using Fierz transformations the two-body 't Hooft interaction in the CCM may be written in the SU(3) limit as [14],

$$H_{\text{'t Hooft}} = \left(-\frac{2}{3}C_s\right) \sum_{i < j} \int d^3r \left[\bar{\psi}(i)\psi(i)\bar{\psi}(j)\psi(j) + \bar{\psi}(i)\gamma_5\psi(i)\bar{\psi}(j)\gamma_5\psi(j) \right. \\ \left. + \bar{\psi}(i)\lambda^a(i)\psi(i)\bar{\psi}(j)\lambda^a(j)\psi(j) + \bar{\psi}(i)\gamma_5\lambda^a(i)\psi(i)\bar{\psi}(j)\gamma_5\lambda^a(j)\psi(j) \right]. \quad (2.27)$$

As mentioned earlier the parameter determining the interaction strength C_s is taken to be 1.6 fm^2 . In the mean field approximation the two-body attractive 't Hooft interaction is ultraviolet divergent due to the fact that quark fields of different flavors are at the same space-time point, and leads to a collapsed baryon. In this work the same regulator as in Kim and Banerjee [14] is used to control this divergence with the same parameter of $r_c = 0.25 \text{ fm}$.

Note that this two-body interaction acts only on flavor anti-symmetric quark pairs. Being a contact interaction the spatial state must be a S state to receive any contribution. Then total spin-flavor-space symmetry requires that the quark pairs be in spin anti-symmetric state. In a flavor decuplet every quark pair is in spin symmetric state and the 't Hooft interaction makes no contribution. However, in an octet quark pairs are in spin anti-symmetric state half the time and the 't Hooft interaction contributes to its mass.⁵ This contribution is attractive, thus lowering the mass of the nucleon relative to that of the Δ , helping to explain the observed $N - \Delta$ mass splitting. It should be noted that the pattern of the mass splitting induced by the 't Hooft interaction is different from the one obtained by using the traditional OGE interaction. The reason is that while the 't Hooft interaction does not contribute when the quark pair is in spin symmetric state, OGE produces a repulsive force $1/9$ in magnitude of the attractive force in the spin anti-symmetric state.

For $N_f = 3$, a three-body interaction among the differently flavored quarks are also possible in addition to the one- and two-body interactions. However, for baryons such a three-body force is absent for the following reason [27,28]. The 't Hooft interaction effectively describes the *change* of the eigenvalue of the zero mode due to the interaction between the quarks and the instanton liquid vacuum. In order to put three quarks in the initial zero mode they must be in a totally anti-symmetric state in flavor and for these quarks to feel the three-body contact interaction the spatial wave function of the baryon must be totally symmetric. Thus the spin wave function for the baryon must be totally anti-symmetric according to total spin-flavor-space symmetry. However this is not possible with spin $1/2$ quarks and the three-body 't Hooft interaction does not contribute for hyperons.⁶

III. DIAGONALIZATION OF H_{BARE}

A. Projection of the Translational Modes

The Hamiltonian used to evaluate the bare baryon masses and three-quark wave functions in this work is given by,

$$H_{\text{Bare}} = H_{\text{Toy}}^{(0)} + H_{\text{'t Hooft}}. \quad (3.1)$$

The matrix elements of H_{Bare} are evaluated in the $j - j$ coupled basis constructed by coupling the total spin $j = l + s$ of individual quarks.⁷ For negative parity baryons they are labeled as $|SU(3), J; SSX\rangle$ where S is a quark in the ground $S_{1/2}$ state and X can be either in the $P \equiv P_{1/2}$ or the $A \equiv P_{3/2}$ state. Before proceeding to the evaluation of the matrix elements of H_{Bare} , it is necessary to identify and project out as much as possible the spurious components in the excited baryon wave functions corresponding to center-of-mass excitations. The prescription used in this work to do this has been used previously [20,37,38] and is strictly valid only in the non-relativistic and SU(6) limits.

⁵ $H_{\text{'t Hooft}}$ also contributes to flavor singlet states as well.

⁶Nevertheless, for two hyperon systems the three-body interaction does not vanish and it has been speculated that such an interaction may even overcome the attractive OGE interaction to unbind the hypothesized H di-baryon [27].

⁷Lower case letters for total spin j , orbital angular momentum l and intrinsic spin s are used to label quarks while upper case letters are used for baryons.

In the non-relativistic limit the spin-flavor wave functions are constructed in the $L - S$ basis by coupling the relative orbital angular momentum of the three-quark system, L , to its total intrinsic spin S . The $L - S$ coupled basis, $|SU(6)^{2S+1}SU(3)_J\rangle$, are labeled by the $SU(6)$ spin-flavor multiplet, the $SU(3)$ flavor multiplet, total angular momentum $J = L + S$ and the spin multiplet $2S + 1$ of the three-quark system. The resulting spin-flavor wave functions of low-lying negative parity baryons belong either to the 56 or the 70 representation of $SU(6)$ spin-flavor. Totally symmetric 56-plet wave functions share the same permutational symmetry as the ground state spin-flavor wave functions. In the non-relativistic quark model with harmonic oscillator confining potential the negative parity 56-plet correspond to the translational mode of the three-quark system. The genuine excitations are represented by permutationally mixed symmetric 70-plet wave functions.

Motivated by this example in the non-relativistic limit, the projection prescription assumes that those linear combinations in the $j - j$ basis corresponding to the 56-plet $L - S$ coupled wave functions are the translational modes of the baryon and is thus spurious. Therefore, H_{Bare} is diagonalized by using only those linear combinations of the $j - j$ basis corresponding to the 70-plet in the $L - S$ basis. Consequently, the spin-flavor compositions of negative parity baryons are described by using the $L - S$ coupled basis of the form $|70,^{2S+1}SU(3)_J\rangle$ as shown in the fourth column of Table III. Also shown in the table are the spin-parity and masses of well-established low-lying negative parity baryons examined in this and the following paper.

Appendix A presents the result of $L - S$ to $j - j$ basis recoupling which is used to express the 56 and 70 components of the $L - S$ basis in terms of the $j - j$ basis. Spurious states are mixed into hyperon wave functions when $SU(6)$ symmetry is broken by $m_s - m_{u,d}$ mass difference. The relative amount of spurious component contained in hyperon wave functions for different values of strange quark masses is estimated in Appendix B. It is found that the spurious components in hyperon wave functions increases with the strange quark mass indicating that this prescription is inapplicable for descriptions of baryons containing a heavy quark [38].

B. Matrix Elements of H_{Bare}

Before presenting the results for bare masses and three-quark wave functions it is instructive to study the matrix elements of $H_{\text{Toy}}^{(0)}$ and $H_{\text{'t Hooft}}$. Since the calculation itself is straightforward, but quite lengthy and not very informative, only the results are discussed in detail. The matrix elements of the Hamiltonian in the $L - S$ coupled basis are labeled as

$$H \equiv \begin{pmatrix} \langle 70,^2Y_J | H | 70,^2Y_J \rangle & \langle 70,^48_J | H | 70,^2Y_J \rangle & \langle 70,^28_J | H | 70,^2Y_J \rangle \\ \langle 70,^48_J | H | 70,^2Y_J \rangle & \langle 70,^48_J | H | 70,^48_J \rangle & \langle 70,^48_J | H | 70,^28_J \rangle \\ \langle 70,^28_J | H | 70,^2Y_J \rangle & \langle 70,^28_J | H | 70,^48_J \rangle & \langle 70,^28_J | H | 70,^28_J \rangle \end{pmatrix}. \quad (3.2)$$

Henceforth the $SU(6)$ spin-flavor label in the $L - S$ basis is dropped and will always be understood to be the 70-plet. Y can be either a flavor singlet for the Λ^* or a decuplet for the Σ^* states. The mass matrices for $J^P = 3/2^-$ and $1/2^-$ N^* states are given by the lower right 2×2 submatrix of Eq. (3.2) involving only the flavor octet matrix elements.

Table IV shows the matrix elements of $H_{\text{Toy}}^{(0)}$ and $H_{\text{'t Hooft}}$ for negative parity nucleon and hyperon resonances. Because the 't Hooft interaction does not contribute to the Δ^* states their bare masses are simply given by the sum of single quark energies plus the constant contribution from the classical χ field energy.⁸ Analytical expressions for the matrix elements of $H_{\text{'t Hooft}}$ are given in Appendix C. The fourth column of the table shows the corresponding matrix elements of the OGE interaction evaluated in the MIT bag model with vanishing quark masses for comparison.

Note that in the $SU(6)$ limit simple patterns emerge among the matrix elements:

In the hyperon sector the model Hamiltonian does not connect flavor singlet or decuplet states with flavor octet states. As a result there are two pure flavor singlet Λ^* states with $J^P = 3/2^-$ and $1/2^-$. Correspondingly, there are two Σ^* states which are purely flavor decuplets with $J^P = 3/2^-$ and $1/2^-$. Since the 't Hooft interaction does not act on flavor decuplets, the bare masses of these two Σ^* states are given by the diagonal matrix elements of $H_{\text{Toy}}^{(0)}$ of 1497 MeV and 1477 MeV, respectively.

These values of decuplet Σ^* masses also correspond to that of $J^P = 3/2^-$ and $1/2^-$ Δ^* baryons resulting in two sets degenerate Σ^* and Δ^* states. However, there are no degenerate partners in the non-strange sector for the purely flavor singlet Λ^* states since no permutationally anti-symmetric three-quark flavor wave function can be constructed with two flavors.

⁸The same applies to the flavor decuplet components of Σ^* states.

The matrix elements of $H_{\text{Toy}}^{(0)}$ and $H_{\text{'t Hooft}}$ involving flavor octet states $|^4 8\rangle_J$ and $|^2 8\rangle_J$ are the same for the N^* , Λ^* and Σ^* baryons sharing the same spin and parity. Thus the bare CCM Hamiltonian produces two sets of degenerate N^* , Λ^* and Σ^* states with $J^P = 3/2^-$ and another two degenerate sets with $J^P = 1/2^-$ in the SU(6) limit.

These mass degeneracies are lifted when single particle energies of strange quarks are corrected for flavor symmetry breaking as discussed in Section II B. However since this correction is applied perturbatively it does not affect the hyperon wave functions.

Note that the off-diagonal matrix elements of $H_{\text{'t Hooft}}$ for $J^P = 3/2^-$ states are negligibly small while those for $J^P = 1/2^-$ states are quite large inducing large mass splittings between the flavor octet states. When compared to the matrix elements of OGE interaction in the MIT bag model one apparent difference can be seen in the decuplet matrix elements involving the two Σ^* states. Moreover, for the flavor singlet matrix elements the 't Hooft interaction gives a much larger and negative value of -90 MeV for the $J^P = 3/2^-$ state compared with -10 MeV for the $J^P = 1/2^-$ matrix element. This pattern is reversed for the OGE interaction where the singlet matrix element of the $J^P = 1/2^-$ state of $\alpha_s(-61)$ MeV is more negative than the $J^P = 3/2^-$ matrix element of $\alpha_s(-12)$ MeV.

C. Bare Masses and Three-Quark Wave Functions

Bare masses and three-quark wave functions of negative parity baryons N^* , Δ^* , Λ^* and Σ^* in the CCM are given in Table V which lists the results before and after including the two-body 't Hooft interaction in the Hamiltonian. With the present set of model parameters the 't Hooft interaction affects the masses of baryon resonances more than their spin-flavor compositions, the latter being determined mostly by single particle energies of valence quarks. The fourth column also shows the bare masses of hyperons after being corrected for flavor symmetry breaking.

1. $J^P = 5/2^-$ States

The matrix elements of $H_{\text{Toy}}^{(0)}$ for $J^P = 5/2^-$ N^* , Λ^* and Σ^* states are all equal and has the value of 1470 MeV. These baryons are purely flavor octet states. In the present quark shell model the three-quark wave functions for $J^P = 5/2^-$ baryons are constructed by exciting one of the ground state quarks from the S to the excited A state. Thus the contribution from single quark energies to $H_{\text{Toy}}^{(0)}$ is simply $2\epsilon_S + \epsilon_A = 1170$ MeV while the classical $\chi^{(0)}$ field energy contributes the remaining 300 MeV. There is no flavor-decuplet $J^P = 5/2^- \Delta^*$ state which is interpreted as a spurious excitation according to the center-of-mass projection prescription as shown in Eq. (A1). The absence of this Δ^* state is in agreement with experiment (see Table III).

Note that numerically, the matrix elements of $H_{\text{Toy}}^{(0)}$ and H_{Bare} are practically equal implying that the effects of 't Hooft interaction on $J^P = 5/2^-$ states are negligibly small. This is because the spin-isospin dependent 't Hooft interaction acts on quark pairs where one of them is in the S and the other in the A state. Being a contact interaction, the resulting interaction energy is expected to be small as verified numerically in this model calculation. Corrections from flavor symmetry breaking lifts the mass degeneracy between the well established $\Lambda(5/2^-)$ and $\Sigma(5/2^-)$ states [40]. However, the resulting mass splitting is only 14 MeV in contrast to the empirical values of 55 MeV. Thus, a further mass splitting of about 40 MeV is necessary from meson exchange corrections.

2. $J^P = 3/2^-$ States

For $J^P = 3/2^-$ baryons there are three sets of degenerate eigenstates of H_{Bare} .

Purely flavor decuplet $\Sigma(3/2^-)_1$ and $\Delta(3/2^-)$ with $E_{\text{Bare}} = 1497$ MeV.

$N(3/2^-)_1$, $\Lambda(3/2^-)_1$ and $\Sigma(3/2^-)_2$ with $E_{\text{Bare}} = 1485$ MeV. The spin-flavor composition of these states is approximately 72 % $|^4 8_{3/2}\rangle$ and 28 % $|^4 8_{3/2}\rangle$. Using the results of $L-S$ to $j-j$ recoupling given in Appendix A, this corresponds to 8% $|8, 3/2; SSA\rangle$, 14% $|8, 3/2; SSA'\rangle$ and 78% $|8, 3/2; SSP\rangle$ in terms of $j-j$ coupled basis. Thus the excited valence quark in these baryons is mostly in the P orbit with about 20% probability of being in the A orbit.

$N(3/2^-)_2, \Lambda(3/2^-)_2$ and $\Sigma(3/2^-)_3$ with $E_{\text{Bare}} = 1439$ MeV. In this set of degenerate states, the excited quark is almost always in the A state with a vanishingly small $|8, 3/2; SSP\rangle$ component. Recall that the single quark energy for the A state ($\epsilon_A = 490$ MeV) is smaller than that of the P state ($\epsilon_P = 550$ MeV). This explains qualitatively why this set of degenerate states has a lower mass than the above set where the excited quark is dominantly in the P orbit.

Finally, the purely flavor singlet $\Lambda(3/2^-)_3$ with $E_{\text{Bare}} = 1380$ MeV has no mass degenerate partners. The values for E_{Bare} for the octet and singlet states are all lower than the values obtained by using H_{Toy} alone indicating that the 't Hooft interaction is attractive for the $J^P = 3/2^-$ baryons.

3. $J^P = 1/2^-$ States

As in the $J^P = 3/2^-$ sector there are three sets of degenerate eigenstates of H_{Bare} for $J^P = 1/2^-$ baryons.

Purely flavor decuplet $\Sigma(1/2^-)_2$ and $\Delta(1/2^-)$ with $E_{\text{Bare}} = 1477$ MeV.

$N(1/2^-)_1, \Lambda(1/2^-)_1$ and $\Sigma(1/2^-)_1$ with $E_{\text{Toy}} = 1529$ MeV and $E_{\text{Bare}} = 1742$ MeV. The spin-flavor composition of these states is about 86% $|^4 8_{1/2}\rangle$ and 14% $|^2 8_{1/2}\rangle$ which translates into 92% $|8, 1/2; SSP\rangle$ and 8% combination of $|8, 1/2; SSA\rangle$ and $|8, 1/2; SSA'\rangle$ states. Thus, the excited quark is dominantly in the P orbit.

$N(1/2^-)_2, \Lambda(1/2^-)_3$ and $\Sigma(1/2^-)_3$ with $E_{\text{Toy}} = 1496$ MeV and $E_{\text{Bare}} = 1450$ MeV. In this set the excited quark is about 45% in the $|8, 1/2; SSP\rangle$ state and 55% in the A state leading to a lower value of energy eigenvalue than the above degenerate set.

Note that for the latter two sets $E_{\text{Bare}} = 1742$ and 1450 MeV, respectively, implying that the 't Hooft interaction induces a large mass splitting among the $J^P = 1/2^-$ baryons. This result can be anticipated by observing that the diagonal matrix elements of $H_{\text{'t Hooft}}$ for the octet $J^P = 1/2^-$ states differ by an order of magnitude as well as by sign (178 and -12 MeV) as shown in Table IV. Finally, the pure flavor-singlet $\Lambda(1/2^-)_2$ state with $E_{\text{Bare}} = 1520$ MeV has no degenerate partners as in the $J^P = 3/2^-$ case.

D. Model comparisons

It is interesting to compare the CCM results for the three-quark wave functions with the results obtained in the MIT bag model using the OGE interaction. Table VI presents the masses and relative percentages of spin-flavor contents of low-lying $J^P = 3/2^-$ and $1/2^-$ Λ^* states in the CCM and in the MIT bag model using respectively the 't Hooft and OGE matrix elements of Table IV.⁹ The same center-of-mass projection prescription is used for both calculations. The Table also presents the results obtained in the NRQM [32] which includes explicit flavor symmetry breaking. These negative parity Λ^* states are chosen since radiative decay widths of the lightest $J^P = 3/2^-$ $\Lambda(1520)$ and $1/2^-$ $\Lambda(1405)$ hyperons are examined in the following paper.

The results for the wave functions of $\Lambda(3/2^-)$ states are qualitatively similar for both the CCM and the MIT bag model, with the lightest $J^P = 3/2^-$ hyperon being a flavor singlet state. Both models predict similar spin-flavor compositions for $\Lambda(3/2^-)_1$ and $\Lambda(3/2^-)_2$. A similar result is also given in the NRQM with the lightest hyperon state having a 81% flavor-singlet component. It should be noted that in the bound state approach to the Skyrme model no bound state solution corresponding to this state exists with physically acceptable values of model parameters [41]. Thus these models suggest that the well-established $\Lambda(1520)$ is a genuine three-quark excited state with a dominant spin-doublet flavor-singlet component.

Differences in model predictions for hyperon wave functions can clearly be seen in the $J^P = 1/2^-$ sector where the only agreement among the three models is the heaviest $\Lambda(1/2^-)_1$, which is mostly a $|^4 8_{1/2}\rangle$ state. In the CCM $\Lambda(1/2^-)_2$ is a pure flavor singlet state while it is a spin-doublet flavor-octet state in the MIT bag model. The NRQM finds $\Lambda(1/2^-)_2$ to be a mixture of 56% $|^2 8_{1/2}\rangle$, 34% $|^4 8_{1/2}\rangle$ and 10% $|^2 1_{1/2}\rangle$. The lightest $J^P = 1/2^-$ hyperon is found to be dominantly a $|^2 8_{1/2}\rangle$ state in the CCM, while it is a pure flavor singlet state in the MIT bag model in the limit of vanishing quark masses. However, when flavor symmetry is broken in the MIT bag model a non-negligible octet

⁹The MIT bag model results were obtained with $B^{1/4} = 145$ MeV, $Z_0 = 0.25$, $\alpha_s = 1.5$ and $m_u = m_d = m_s = 0$ MeV.

component is mixed into the $\Lambda(1/2^-)_3$ wave function [4,20]. In the NRQM this hyperon resonance is predominantly a flavor singlet state. In previous works on the K^-p atom and $\bar{K}N$ scattering in the cloudy bag model [42,43], it has been assumed that the $\Lambda(1405)$ is a pure flavor singlet. The present example indicates that the cloudy bag model Hamiltonian must be properly diagonalized before neglecting the flavor octet component of the $\Lambda(1405)$ wave function.

IV. MESON-BARYON VERTEX FUNCTIONS

In this section three-quark wave functions obtained by diagonalizing H_{Bare} are used to calculate meson-excited bare baryon vertex functions to lowest order in meson-quark coupling. Each of these vertex functions is a product of a coupling constant and a form factor when the baryons are on their mass-shell. Meson-bare baryon form factors for negative parity resonances are extracted by applying a method originally introduced for ground states [44,45]. It will be shown that model predictions for the form factors have problems with their analytical structures and, consequently, only their values at zero three-momentum transfer are reliable. As examples, axial coupling constants for excited nucleons are determined using the Goldberger-Trieman relation.

The equation of motion for the pseudoscalar meson fields $\phi^a(\vec{r})$ corresponding to the SU(3) CCM Lagrangian given in Eq. (2.6) is,

$$(\square + m_\phi^2) \phi^a(\vec{r}) = \frac{g_\phi}{(g_\chi \chi^{(0)}(r))^2} \bar{\psi}(\vec{r}) \left(i\lambda^a \gamma_5 \right) \psi(\vec{r}) \equiv J_5^a(\vec{r})_q. \quad (4.1)$$

In the present work, where the excited quark is either in the P or the A state, the meson-quark source current is given by

$$J_5^a(\vec{r})_q = \frac{g_\phi}{(g_\chi \chi^{(0)}(r))^2} \left[\bar{u}_S(\vec{r}) \left(i\lambda^a \gamma_5 \right) u_S(\vec{r}) + \bar{u}_P(\vec{r}) \left(i\lambda^a \gamma_5 \right) u_P(\vec{r}) \right. \\ \left. + \bar{u}_A(\vec{r}) \left(i\lambda^a \gamma_5 \right) u_A(\vec{r}) + \bar{u}_P(\vec{r}) \left(i\lambda^a \gamma_5 \right) u_A(\vec{r}) \right] \quad (4.2)$$

where the quark spinors for states S , P and A are given in Eqs. (2.15) to (2.17), respectively.

To lowest order in meson-quark coupling, the meson-excited bare baryon vertex function is given by the matrix element of the Fourier transform of Eq. (4.2) in the static limit,

$$V(BB'\phi)(|\vec{q}|) \equiv \sum_{j=1}^3 \langle (qqq)_{B'} | \int d^3r e^{+i\vec{q}\cdot\vec{r}} J_5^a(\vec{r})_q^j | (qqq)_B \rangle, \quad (4.3)$$

where $|(qqq)_B\rangle$ is a three-quark state corresponding to baryon B . On the right side of Eq. (4.3), summation over the quark index j is shown explicitly for the single quark operator $J_5^a(\vec{r})_q^j$. In general, the initial and final baryons, B and B' , may belong to different spin-parity states. However, in this work only those vertex functions involving initial and final states with the same J^P value are considered. This means that when using $V(BB'\phi)$ to calculate one-meson loop contributions to baryon observables, the intermediate baryon state is restricted to share the same spin-parity as the initial and final baryons.

The Fourier transformed meson-quark source current operator may be written as,

$$\int d^3r e^{+i\vec{q}\cdot\vec{r}} J_5^a(\vec{r})_q^j = F_{SS}^{l=1}(|\vec{q}|) \mathcal{O}_{SS}^{a,l=1}(\hat{q})^j + F_{PP}^{l=1}(|\vec{q}|) \mathcal{O}_{PP}^{a,l=1}(\hat{q})^j + F_{AA}^{l=1}(|\vec{q}|) \mathcal{O}_{AA}^{a,l=1}(\hat{q})^j \\ + F_{PA}^{l=1}(|\vec{q}|) \mathcal{O}_{PA}^{a,l=1}(\hat{q})^j + F_{AA}^{l=3}(|\vec{q}|) \mathcal{O}_{AA}^{a,l=3}(\hat{q})^j. \quad (4.4)$$

Here $\mathcal{O}_{MN}^{a,l}(\hat{q})^j$ is the l -th multipole single quark operator acting on the spin-flavor space of the j -th quark. The subscripts MN identifies various contributions from the meson-quark source current, Eq (4.2), in an obvious manner. These operators and their corresponding functions $F_{MN}^l(|\vec{q}|)$ are defined in Appendix D. If the initial and final baryon states are restricted to share the same J^P value, the operators shown in Eq. (4.4) are the only possible multipole contributions in the present model with a quark shell model space consisting only of S , P and A states.

Meson-baryon form factors are determined by defining a meson-baryon interaction at the *hadronic* level and equating the Fourier transformed meson-quark and meson-baryon source currents [44,45]. For spin 1/2 baryons assume a meson-baryon interaction of the form

$$\mathcal{L}_{\phi B'B} = g_{\phi B'B} \bar{\Psi}(\vec{r})_{B'} (i\phi^a \lambda^a \gamma_5) \Psi(\vec{r})_B, \quad (4.5)$$

where $\Psi(\vec{r})_B$ is a single baryon field with total spin J and parity P . The corresponding meson–baryon source current $J_5^a(\vec{r})_B$ is

$$J_5^a(\vec{r})_B \equiv g_{\phi B'B} \bar{\Psi}(\vec{r})_{B'} \left(i\lambda^a \gamma_5 \right) \Psi(\vec{r})_B. \quad (4.6)$$

For example, the $BB'\phi$ form factor $G_{BB'\phi}(k)$ for ground state octet baryons B and B' with $|J^P, J_z\rangle = |\frac{1}{2}^+, +\frac{1}{2}\rangle$ is determined as follows:

$$\begin{aligned} \langle B'(1/2^+, +1/2) | \int d^3r e^{+i\vec{q}\cdot\vec{r}} J_5^a(\vec{r})_B | B(1/2^+, +1/2) \rangle \\ = -iG_{BB'\phi}^{l=1}(|\vec{q}|) \frac{|\vec{q}|}{2M_N} \langle B'(1/2^+, +1/2) | \lambda^a \hat{q} \cdot \vec{\sigma} | B(1/2^+, +1/2) \rangle \\ = -iG_{BB'\phi}^{l=1}(|\vec{q}|) \frac{|\vec{q}|}{2M_N} \langle B' | \lambda^a | B \rangle \\ = \sum_{j=1}^3 \langle (qqq)_{B'} | \int d^3r e^{+i\vec{q}\cdot\vec{r}} J_5^a(\vec{r})_q^j | (qqq)_B \rangle \\ = \sum_{j=1}^3 \langle (qqq)_{B'} | F_{SS}^{l=1}(|\vec{q}|) \mathcal{O}_{SS}^{a,l=1}(\hat{q})^j | (qqq)_B \rangle. \end{aligned} \quad (4.7)$$

The superscript in $G_{BB'\phi}^{l=1}$ indicates that it is a dipole form factor. Note that by definition $G_{BB'\phi}^{l=1}$ is divided by twice the nucleon mass $2M_N$ regardless of the types of initial and final baryons B and B' . This convention, which differs from the one given in [45] for the $N\Delta\pi$ form factor, will be extended to extract excited baryon form factors.

For excited baryons with $|J^P, J_z\rangle = |\frac{5}{2}^-, +\frac{5}{2}\rangle, |\frac{3}{2}^-, +\frac{3}{2}\rangle$, and $|\frac{1}{2}^-, +\frac{1}{2}\rangle$, it is found that only dipole form factors are numerically significant.¹⁰ They are given by

$$\begin{aligned} \underline{J^P = 5/2^-} : \quad & -iG_{BB'\phi}^{l=1}(|\vec{q}|) \frac{|\vec{q}|}{2M_N} \left(\frac{1}{17} \right) \langle B' | \lambda^a | B \rangle = \\ & \sum_{j=1}^3 \langle (qqq)_{B'} | F_{SS}^{l=1}(|\vec{q}|) \mathcal{O}_{SS}^{a,l=1}(\hat{q})^j + F_{AA}^{l=1}(|\vec{q}|) \mathcal{O}_{AA}^{a,l=1}(\hat{q})^j | (qqq)_B \rangle \end{aligned} \quad (4.8)$$

$$\begin{aligned} \underline{J^P = 3/2^-} : \quad & -iG_{BB'\phi}^{l=1}(|\vec{q}|) \frac{|\vec{q}|}{2M_N} \left(\frac{1}{5} \right) \langle B' | \lambda^a | B \rangle = \\ & \sum_{j=1}^3 \langle (qqq)_{B'} | F_{SS}^{l=1}(|\vec{q}|) \mathcal{O}_{SS}^{a,l=1}(\hat{q})^j + F_{PP}^{l=1}(|\vec{q}|) \mathcal{O}_{PP}^{a,l=1}(\hat{q})^j \\ & \quad + F_{AA}^{l=1}(|\vec{q}|) \mathcal{O}_{AA}^{a,l=1}(\hat{q})^j + F_{AP}^{l=1}(|\vec{q}|) \mathcal{O}_{AP}^{a,l=1}(\hat{q})^j | (qqq)_B \rangle \end{aligned} \quad (4.9)$$

$$\begin{aligned} \underline{J^P = 1/2^-} : \quad & -iG_{BB'\phi}^{l=1}(|\vec{q}|) \frac{|\vec{q}|}{2M_N} \langle B' | \lambda^a | B \rangle = \\ & \sum_{j=1}^3 \langle (qqq)_{B'} | F_{SS}^{l=1}(|\vec{q}|) \mathcal{O}_{SS}^{a,l=1}(\hat{q})^j + F_{PP}^{l=1}(|\vec{q}|) \mathcal{O}_{PP}^{a,l=1}(\hat{q})^j \\ & \quad + F_{AA}^{l=1}(|\vec{q}|) \mathcal{O}_{AA}^{a,l=1}(\hat{q})^j + F_{AP}^{l=1}(|\vec{q}|) \mathcal{O}_{AP}^{a,l=1}(\hat{q})^j | (qqq)_B \rangle \end{aligned} \quad (4.10)$$

Figure 2 shows the normalized form factors $R(N^*N^*\pi)(|\vec{q}|) \equiv G_{N^*N^*\pi}^{l=1}(|\vec{q}|)/G_{N^*N^*\pi}^{l=1}(0)$ for negative parity nucleons under consideration. It can be seen from the figure that all the $N^*N^*\pi$ form factors are qualitatively similar to lowest

¹⁰Form factors involving baryons belonging to different flavor multiplets may be obtained by exploiting the assumed SU(3) symmetry [44,45].

order in meson–quark coupling. In particular, the three momentum cut–off of each of the form factors is around $|\vec{q}| = 1200$ MeV. In the hyperon sector, a similar $|\vec{q}|$ dependence can also be seen in the normalized dipole form factors involving the $J^P = 3/2^-$ flavor singlet and N^* states, $R(\Lambda_1 N^* k)(|\vec{q}|)$, as shown in Figure 3a. However, the corresponding form factors involving the $J^P = 1/2^-$ states, shown in Figure 3b, are quite different. In this case the $\Lambda_1 N^* k$ form factors change sign for the first time around $|\vec{q}| = 400$ MeV and encounter a second node at about $|\vec{q}| = 1200$ MeV. As the figure indicates these two form factors are proportional to each other since the Fourier transformed meson–quark source current only connects the flavor singlet $\Lambda(1/2^-)_1$ to the spin–doublet flavor–octet (${}^2 8_{1/2}$) component of the excited nucleon $N(1/2^-)$.

Having extracted the meson–baryon form factors from meson–quark vertex functions it is tempting to calculate the meson–baryon coupling constants. However it should be emphasized that the model form factors presented in this section do not have the correct behaviour when analytically continued to the complex $|\vec{q}|$ –plane. The analytically continued CCM form factors are entire functions on the *whole complex plane* and not the cut plane with a branch point at the normal threshold of $|\vec{q}|^2 = 4M_B^2$ [46]. In particular, there are no dynamical singularities in the CCM form factors which arise from summing over intermediate states in the dispersion representation. Thus the $|\vec{q}|$ dependencies of these form factors are questionable and the only reliable quantity to extract from these form factors are their values at $|\vec{q}| = 0$, such as $G_{NN\pi}(0)$ to be discussed below. In order to evaluate the meson–baryon coupling constants the value of the form factor at $|\vec{q}|^2 = m_\phi^2$ is required. For pion–baryon coupling constants it might be reasonable to Taylor expand the form factor around $|\vec{q}| = 0$ since the pion mass is small,

$$G_{BB'\pi}(m_\pi^2) = G_{BB'\pi}(0) + m_\pi^2 \frac{d}{d|\vec{q}|^2} G_{BB'\pi}(|\vec{q}|^2)|_{|\vec{q}|^2=0} + \dots \quad (4.11)$$

However this expansion would certainly not be justified for kaons and other heavier mesons. It should be remarked that this problem is shared by other relativistic quark models with confinement and meson–quark coupling, such as the chiral bag model [44].

When $B = B' = N(1/2^+)$ and $\phi = \pi$, the value of the CCM hadronic form factor at $|\vec{q}| = 0$ is given by $G_{N(1/2^+)N(1/2^+)\pi}^{l=1}(0) = 12.1$. For excited nucleons the values of dipole form factors $G_{N^*N^*\pi}^{l=1}(|\vec{q}|)$ at zero three momentum transfers are found to be

$$G_{N(5/2^-)N(5/2^-)\pi}^{l=1}(0) = 157.5 \quad (4.12)$$

$$G_{N(3/2^-)_1 N(3/2^-)_1 \pi}^{l=1}(0) = 41.3 \quad G_{N(3/2^-)_2 N(3/2^-)_2 \pi}^{l=1}(0) = -16.6 \quad (4.13)$$

$$G_{N(1/2^-)_1 N(1/2^-)_1 \pi}^{l=1}(0) = 8.4 \quad G_{N(1/2^-)_2 N(1/2^-)_2 \pi}^{l=1}(0) = -2.4. \quad (4.14)$$

The reason for large values of $G_{N(5/2^-)N(5/2^-)\pi}^{l=1}(0)$ and $G_{N(3/2^-)_1 N(3/2^-)_1 \pi}^{l=1}(0)$ are due to the spin matrix elements of $(1/17)$ and $(1/5)$ appearing on the left sides of Eqs. (4.8) and (4.9), respectively.

If the Goldberger–Trieman relation

$$g_A(0) = \frac{f_\pi}{M_N} G_{NN\pi}(0) \quad (4.15)$$

is applied to the ground state form factor $G_{N(1/2^+)N(1/2^+)\pi}^{l=1}$, the resulting axial coupling constant is $g_A \approx 1.21$ which is close to the empirical value of $g_A \approx 1.26$. Note that in the derivation of Eq. (4.15), the free Dirac equation is used to introduce the nucleon mass M_N . When this relation is extended to excited states, the nucleon mass M_N in Eq. (4.15) should be replaced with the excited nucleon masses M_N^* , *i.e.*

$$g_A(0) = \frac{f_\pi}{M_N^*} G_{N^*N^*\pi}(0). \quad (4.16)$$

Thus to lowest order in meson–quark coupling, the axial coupling constants for the excited nucleons are

$$g_A^{N(5/2^-)} = 8.74 \quad (4.17)$$

$$g_A^{N(3/2^-)_1} = 2.3 \quad g_A^{N(3/2^-)_2} = -1.0 \quad (4.18)$$

$$g_A^{N(1/2^-)_1} = 0.47 \quad g_A^{N(1/2^-)_2} = -0.15 \quad (4.19)$$

V. DISCUSSION

In this paper a toy model version of CCM is used together with the classical 't Hooft interaction to calculate the quark contribution to the masses and wave functions of low-lying negative parity baryons N^* , Δ^* , Λ^* and Σ^* . A baryon in the CCM is a composite object consisting of interacting quarks, mesons and a chiral singlet χ field confined inside a dynamically generated region. The meson fields in this model are effective degrees of freedom describing the exchange of a tower of gluons between quarks. In the mean field approximation which is used in this work, meson exchanges between quarks are treated perturbatively at the quantum level. Hence in the mean-field theory there is a “bare” baryon with its wave function determined by the valence quark dynamics which couples to mesons yielding a physical baryon state. Mesonic exchange corrections to bare baryon properties will be examined in a following paper.

Masses and three-quark wave functions of bare excited baryon states are calculated by diagonalizing H_{Bare} defined in Eq. (3.1) using $m_u = m_d = m_s = 7.5$ MeV. A prescription, strictly valid only in the SU(6) and non-relativistic limits, is used to approximately eliminate the spurious degrees of freedom associated with the center-of-mass motion of the baryon. A simple test showed that this prescription becomes less reliable with increasing strange quark mass. It is found that the 't Hooft interaction affects the bare masses of negative parity baryons more than their wave functions which are determined mostly by single quark energies. The CCM results for the bare masses indicate that mesonic corrections to excited baryon masses must play an important role in order to reproduce the observed mass splittings. It should be remarked that corrections to bare masses and wave functions due to flavor symmetry breaking should be investigated.

Using the bare baryon wave functions meson-excited baryon vertex functions have been calculated to lowest order in meson-quark coupling. Meson-baryon form factors for negative parity baryons were extracted from vertex functions by extending the conventional definition used for ground states. However, it is emphasized that because the model form factors do not have the correct behaviour when analytically continued to the complex plane, their momentum dependencies are questionable and only their values at zero momentum transfer can be trusted. By applying the Goldberger-Trieman relationship the quark contribution to weak axial charges for negative parity nucleons have been determined. In a following paper the vertex functions are used to regulated mesonic corrections to bare baryon properties in the one-loop and heavy baryon approximations. The second paper also examines the four radiative decays of the two lightest Λ^* hyperons which will be measured in an upcoming Jefferson Lab experiment [1], and will serve as an excellent testing ground for *all* models of baryon structure.

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APPENDIX A: $L - S$ TO $J - J$ BASIS RECOUPLING

As discussed in Section III A, the center-of-mass projection prescription used in this work assumes that the spurious components in negative parity baryon wave functions are described by those linear combinations in the $j - j$ basis corresponding to the 56-plet in the $L - S$ basis [20]. The remaining linear combinations corresponding to the 70-plet are used to diagonalize the model Hamiltonian. This Appendix presents the results of $L - S$ to $j - j$ recoupling using the notations for $L - S$ and $j - j$ coupled basis defined in Section III A.

$$\begin{aligned} \underline{J^P = 5/2^-} \\ |56, {}^4 10_{5/2}\rangle &= |10, 5/2; SSA\rangle \end{aligned} \tag{A1}$$

$$|70, {}^4 8_{5/2}\rangle = |8, 5/2; SSA\rangle \tag{A2}$$

$$\begin{aligned} \underline{J^P = 3/2^-} \\ |70, {}^2 1_{3/2}\rangle &= -|1, 3/2; SSA\rangle' \end{aligned} \tag{A3}$$

$$\begin{pmatrix} |56, {}^4 10_{3/2}\rangle \\ |70, {}^2 10_{3/2}\rangle \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & \frac{\sqrt{5}}{3} \\ \frac{\sqrt{5}}{3} & -\frac{2}{3} \end{pmatrix} \begin{pmatrix} |10, 3/2; SSA\rangle \\ |10, 3/2; SSP\rangle \end{pmatrix} \tag{A4}$$

$$\begin{pmatrix} |56,^2 8_{3/2}\rangle \\ |70,^4 8_{3/2}\rangle \\ |70,^2 8_{3/2}\rangle \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{5}{18}} & \frac{1}{\sqrt{2}} & -\frac{\sqrt{2}}{3} \\ \frac{2}{3} & 0 & \frac{\sqrt{5}}{3} \\ -\sqrt{\frac{5}{18}} & \frac{1}{\sqrt{2}} & \frac{\sqrt{2}}{3} \end{pmatrix} \begin{pmatrix} |8, 3/2; SSA\rangle \\ |8, 3/2; SSA'\rangle \\ |8, 3/2; SSP\rangle \end{pmatrix} \quad (\text{A5})$$

$$\begin{aligned} J^P = 1/2^- \\ |70,^2 1_{1/2}\rangle = -|1, 1/2; SSP'\rangle \end{aligned} \quad (\text{A6})$$

$$\begin{pmatrix} |56,^4 10_{1/2}\rangle \\ |70,^2 10_{1/2}\rangle \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & \frac{\sqrt{8}}{3} \\ \frac{\sqrt{8}}{3} & -\frac{1}{3} \end{pmatrix} \begin{pmatrix} |10, 1/2; SSA\rangle \\ |10, 1/2; SSP\rangle \end{pmatrix} \quad (\text{A7})$$

$$\begin{pmatrix} |56,^2 8_{1/2}\rangle \\ |70,^4 8_{1/2}\rangle \\ |70,^2 8_{1/2}\rangle \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & \frac{1}{\sqrt{2}} & -\frac{\sqrt{2}}{6} \\ \frac{1}{3} & 0 & \frac{\sqrt{8}}{3} \\ -\frac{2}{3} & \frac{1}{\sqrt{2}} & \frac{\sqrt{2}}{6} \end{pmatrix} \begin{pmatrix} |8, 1/2; SSA\rangle \\ |8, 1/2; SSP'\rangle \\ |8, 1/2; SSP\rangle \end{pmatrix} \quad (\text{A8})$$

APPENDIX B: A TEST FOR THE PROJECTION PRESCRIPTION

Because the projection prescription assumes SU(6) symmetry, the matrix elements of H_{Bare} , Eq. (3.1), are evaluated using $m_s = m_{u,d}$. Corrections from flavor symmetry breaking ($m_s \neq m_{u,d}$) to neative parity hyperon masses are given to first order as discussed in Section II B. However, the breaking of flavor symmetry introduces spurious components in excited hyperon wave functions. This Appendix presents an estimate of the relative amount of spurious component introduced in excited hyperon wave functions for different values of strange quark mass using the well-established $J^P = 5/2^-$ $\Sigma(1775)$ resonance as an example.

According to the projection prescription the wave function of this hyperon resonance, $|\Sigma(1775)\rangle \equiv |\Sigma^*\rangle_{\text{ex}}$, is a pure flavor octet state identified with the 70-plet in the $L-S$ coupled basis as shown in Eq. (A2). The flavor decuplet state given in Eq. (A1) is assumed to be a spurious state, $|\Sigma^*\rangle_{\text{sp}}$, corresponding to the translational mode of the baryon. In the $j-j$ coupled basis these two orthogonal states are given by

$$|\Sigma^*\rangle_{\text{ex}} = |8, 5/2; SSA\rangle = \sqrt{\frac{2}{3}}|\Sigma^*\rangle_1 - \sqrt{\frac{1}{3}}|\Sigma^*\rangle_2, \quad (\text{B1})$$

$$|\Sigma^*\rangle_{\text{sp}} = |10, 5/2; SSA\rangle = \sqrt{\frac{1}{3}}|\Sigma^*\rangle_1 + \sqrt{\frac{2}{3}}|\Sigma^*\rangle_2, \quad (\text{B2})$$

where,

$$|\Sigma^*\rangle_1 \equiv \frac{1}{\sqrt{6}}(u\uparrow d\uparrow s\alpha + d\uparrow u\uparrow s\alpha + u\uparrow s\alpha d\uparrow + d\uparrow s\alpha u\uparrow + s\alpha u\uparrow d\uparrow + s\alpha d\uparrow u\uparrow), \quad (\text{B3})$$

$$\begin{aligned} |\Sigma^*\rangle_2 \equiv \frac{1}{\sqrt{12}}(u\uparrow d\alpha s\uparrow + d\alpha u\uparrow s\uparrow + u\uparrow s\uparrow d\alpha + d\alpha s\uparrow u\uparrow + s\uparrow u\uparrow d\alpha + s\uparrow d\alpha u\uparrow \\ + u\alpha d\uparrow s\uparrow + d\uparrow u\alpha s\uparrow + u\alpha s\uparrow d\uparrow + d\uparrow s\uparrow u\alpha + s\uparrow u\alpha d\uparrow + s\uparrow d\uparrow u\alpha). \end{aligned} \quad (\text{B4})$$

The spin states are defined as $\uparrow \equiv |j = 1/2, j_z = +1/2\rangle$ and $\alpha \equiv |j = 3/2, j_z = +3/2\rangle$. Note that in Eq. (B1) the strange quark is in the excited A state with a probability of two-thirds, while this number is lowered to one-third for the spurious $|\Sigma^*\rangle_{\text{sp}}$ state shown in Eq. (B2). The SU(6) spin-flavor wave function for the ground state hyperon $J^P = 1/2^+$ $\Sigma(1192)$, denoted as $|\Sigma^0\rangle_{\text{gs}}$, is

$$|\Sigma^0\rangle_{\text{gs}} = \sqrt{\frac{2}{3}}|\Sigma^0\rangle_1 - \sqrt{\frac{1}{3}}|\Sigma^0\rangle_2, \quad (\text{B5})$$

where,

$$|\Sigma^0\rangle_1 \equiv \frac{1}{\sqrt{6}}(u\uparrow d\uparrow s\downarrow + d\uparrow u\uparrow s\downarrow + u\uparrow s\downarrow d\uparrow + d\uparrow s\downarrow u\uparrow + s\downarrow u\uparrow d\uparrow + s\downarrow d\uparrow u\uparrow), \quad (\text{B6})$$

$$\begin{aligned} |\Sigma^0\rangle_2 \equiv \frac{1}{\sqrt{12}}(u\uparrow d\downarrow s\uparrow + d\downarrow u\uparrow s\uparrow + u\uparrow s\uparrow d\downarrow + d\downarrow s\uparrow u\uparrow + s\uparrow u\uparrow d\downarrow + s\uparrow d\downarrow u\uparrow \\ + u\downarrow d\uparrow s\uparrow + d\uparrow u\downarrow s\uparrow + u\downarrow s\uparrow d\uparrow + d\uparrow s\uparrow u\downarrow + s\uparrow u\downarrow d\uparrow + s\uparrow d\uparrow u\downarrow), \end{aligned} \quad (\text{B7})$$

with $\downarrow \equiv |j = 1/2, j_z = -1/2\rangle$.

Let \mathcal{O} be some operator exciting the internal degrees of freedom of the three-quark system so that $\mathcal{O}|\Sigma^0\rangle_{\text{gs}}$ is proportional to $|\Sigma^*\rangle_{\text{ex}}$. Then the following relations hold between the ground and excited state wave functions in the exact SU(6) limit

$$\text{ex}\langle\Sigma^*|\mathcal{O}|\Sigma^0\rangle_{\text{gs}} \neq 0, \quad (\text{B8})$$

$$\text{sp}\langle\Sigma^*|\mathcal{O}|\Sigma^0\rangle_{\text{gs}} = 0. \quad (\text{B9})$$

The deviation from the vanishing value of Eq. (B9) is used to estimate the amount of spurious component introduced due to flavor symmetry breaking as follows.

The excitation operator \mathcal{O} may be constructed by considering two single particle operators S_μ^{SS} and L_μ^{AS} defined as

$$S_\mu^{SS} \equiv \int d^3r \bar{u}_S(\vec{r}) \gamma_5 \gamma_\mu u_S(\vec{r}) \quad (\text{B10})$$

$$L_\mu^{AS} \equiv \int d^3r \bar{u}_A(\vec{r}) r_\mu u_S(\vec{r}) \quad (\text{B11})$$

where u_S and u_A are the quark spinors for states S and A shown in Eqs. (2.15) and (2.17), respectively. Let $S_+ \equiv (S_1^{SS} + iS_2^{SS})/2$ and $L_+ \equiv (L_1^{AS} + iL_2^{AS})/2$. The operator L_+S_+ acts on the spin-flavor space of a quark in the following way,

$$L_+S_+|u\uparrow\rangle = L_+S_+|d\uparrow\rangle = L_+S_+|s\uparrow\rangle = 0, \quad (\text{B12})$$

and

$$L_+S_+|u\downarrow\rangle = L_0S_0|u\alpha\rangle; \quad L_+S_+|d\downarrow\rangle = L_0S_0|d\alpha\rangle; \quad L_+S_+|s\downarrow\rangle = L_mS_m|s\alpha\rangle. \quad (\text{B13})$$

In Eq. (B13) S_0 , L_0 , S_m and L_m are defined as

$$S_{0/m} \equiv 4\pi \int dr r^2 \left(G_0^2(r) + \frac{1}{3} F_0^2(r) \right)_{0/m}, \quad (\text{B14})$$

$$L_{0/m} \equiv \frac{4\pi}{3\sqrt{2}} \int dr r^3 \left(-G_0(r)G_3(r) + F_0(r)F_3(r) \right)_{0/m}. \quad (\text{B15})$$

The value of the up and down quark masses used to obtain the radial functions G_i and F_i for S_0 and L_0 is 7.5 MeV, while the strange quark mass has been left as a parameter for S_m and L_m .

Thus the operator L_+S_+ raises the total angular momentum j of a quark of given flavor from \downarrow to α while annihilating a single quark state with $j = \uparrow$. Using L_+S_+ the excitation operator \mathcal{O} is constructed as

$$\mathcal{O} \equiv \sum_{k=1}^3 L_+(k) S_+(k), \quad (\text{B16})$$

where k is the quark index. When \mathcal{O} acts on the ground state wave function $|\Sigma^0\rangle_{\text{gs}}$ it excites quarks with $j = \downarrow$ to α thus creating either the $J^P = 5/2^-$ $|\Sigma^*\rangle_{\text{ex}}$ or $|\Sigma^*\rangle_{\text{sp}}$ state where the excited quark must be in the A state.

Taking $m_s - m_{u,d}$ mass difference into account, the matrix elements corresponding to Eqs. (B8) and (B9) are found to be

$$\text{ex}\langle\Sigma^*|\mathcal{O}|\Sigma^0\rangle_{\text{gs}} = \frac{2}{3}L_mS_m + \frac{1}{3}L_0S_0, \quad (\text{B17})$$

$$\text{sp}\langle\Sigma^*|\mathcal{O}|\Sigma^0\rangle_{\text{gs}} = \frac{\sqrt{2}}{3} \left(L_mS_m - L_0S_0 \right). \quad (\text{B18})$$

When $m_s = m_{u,d}$, $L_m \rightarrow L_0$ and $S_m \rightarrow S_0$. Consequently, Eqs. (B8) and (B9) hold exactly in the SU(6) limit and $|\Sigma^*\rangle_{\text{ex}}$ is interpreted as the genuinely excited $|\Sigma(1775)\rangle$ state. However when $m_s \neq m_{u,d}$, $L_mS_m - L_0S_0 \neq 0$ and Eq. (B18) no longer vanishes indicating the presence of spurious components in the excited $\mathcal{O}|\Sigma^0\rangle_{\text{gs}}$ state.

The difference $\delta LS \equiv |L_mS_m - L_0S_0|$, which may be calculated in any relativistic quark model, is used as an estimate of theoretical uncertainty arising from $m_s - m_{u,d}$ mass difference. The values for δLS are found to be 0.151 and 0.206 for $m_s = 200$ and 300 MeV, respectively, indicating that the projection prescription becomes less applicable with increasing strange quark mass. Similar conclusion is presented in [38] where it is remarked that MIT bag model and its chirally extended versions are not suitable for describing a baryon with one heavy quark. It should be remarked that the projection prescription assumes that the angular momentum of a quark spinor is given by its upper component and this assumption contributes to the model uncertainty even in the SU(6) limit.

APPENDIX C: 't HOOFT INTERACTION MATRIX ELEMENTS

This Appendix presents the matrix elements of $H_{\text{'t Hooft}}$, Eq. (2.27), used in calculating the bare masses of low-lying negative parity baryons. The matrix elements have been evaluated assuming $m_s = m_{u,d}$ and are to be multiplied by an universal factor of $-\frac{16\pi}{3}C_S$ with $C_S = 1.6 \text{ fm}^2$ [14,24]. As discussed in Section II C, the 't Hooft interaction needs to be regularized. The regulator used in this work is

$$R(r) = 1 - e^{(r/r_c)^2}, \quad (\text{C1})$$

which is the same as in Kim and Banerjee [14] with the same parameter $r_c = 0.25 \text{ fm}$. The notation for the $j - j$ coupled basis are defined in Section III A. Recall that since the two-body 't Hooft interaction operates on flavor anti-symmetric quark pairs, the Δ^* states and flavor decuplet components of Σ^* are unaffected by this interaction. Also because of assumed flavor symmetry, the matrix elements involving the octet components of N^* , Λ^* and Σ^* are equal for given spin and parity.

$$\underline{J^P = 5/2^- : N^*/\Lambda^*/\Sigma^*}$$

$$\langle 8, 5/2; SSA | H | 8, 5/2; SSA \rangle = \frac{3}{2} F1 \quad (\text{C2})$$

$$\underline{J^P = 3/2^- : N^*/\Lambda^*/\Sigma^*}$$

$$\langle 1, 3/2; SSA | H | 1, 3/2; SSA \rangle' = F1 + F3 + F5 \quad (\text{C3})$$

$$\langle 8, 3/2; SSA | H | 8, 3/2; SSA \rangle = \frac{3}{5} \left(\frac{3}{4} F1 - \sqrt{3} F2 + \frac{3}{4} F3 + F4 \right) \quad (\text{C4})$$

$$\langle 8, 3/2; SSA | H | 8, 3/2; SSA \rangle' = \frac{1}{4} (F1 + F3 + 4F5) \quad (\text{C5})$$

$$\langle 8, 3/2; SSP | H | 8, 3/2; SSP \rangle = \frac{3}{2} F6 \quad (\text{C6})$$

$$\langle 8, 3/2; SSA | H | 8, 3/2; SSA \rangle = \frac{3}{2\sqrt{5}} \left(-\frac{1}{2} F1 - \frac{1}{\sqrt{3}} F2 + \frac{1}{2} F3 \right) \quad (\text{C7})$$

$$\langle 8, 3/2; SSP | H | 8, 3/2; SSA \rangle = \frac{3}{\sqrt{10}} \left(-\frac{\sqrt{3}}{2} F7 + F8 \right) \quad (\text{C8})$$

$$\langle 8, 3/2; SSP | H | 8, 3/2; SSA \rangle' = -\frac{3}{2\sqrt{6}} F7 \quad (\text{C9})$$

$$\underline{J^P = 1/2^- : N^*/\Lambda^*/\Sigma^*}$$

$$\langle 1, 1/2; SSP | H | 1, 1/2; SSP \rangle' = F5 + F6 + F13 \quad (\text{C10})$$

$$\langle 8, 1/2; SSP | H | 8, 1/2; SSP \rangle = \frac{1}{4} F6 + F11 - F12 + \frac{1}{4} F13 \quad (\text{C11})$$

$$\langle 8, 1/2; SSP | H | 8, 1/2; SSP \rangle' = \frac{1}{4} (4F5 + F6 + F13) \quad (\text{C12})$$

$$\langle 8, 1/2; SSA | H | 8, 1/2; SSA \rangle = \frac{3}{2} \left(-\frac{1}{\sqrt{3}} F2 + \frac{1}{2} F3 + \frac{1}{6} F4 + \frac{1}{3} F9 - \frac{1}{3} F10 \right) \quad (\text{C13})$$

$$\langle 8, 1/2; SSP | H | 8, 1/2; SSP \rangle' = \frac{1}{2} \left(-\frac{1}{2} F6 - F12 + \frac{1}{2} F13 \right) \quad (\text{C14})$$

$$\langle 8, 1/2; SSA | H | 8, 1/2; SSP \rangle = \frac{1}{2} \left(-\frac{\sqrt{3}}{2} F7 + \frac{1}{2} F8 - 2F14 - \frac{1}{2} F15 \right) \quad (\text{C15})$$

$$\langle 8, 1/2; SSA | H | 8, 1/2; SSP \rangle' = \frac{\sqrt{3}}{4} \left(F7 - \frac{1}{\sqrt{3}} F8 + \frac{1}{\sqrt{3}} F15 \right) \quad (\text{C16})$$

The functions F1 to F15 are given by

$$F1 = -\frac{8}{15} \int dr R r^2 \left(G_0 F_3 - G_3 F_0 \right)^2 \quad (C17)$$

$$F2 = -\frac{2}{3\sqrt{3}} \int dr R r^2 \left[\frac{8}{5} \left(G_0 G_3 F_0 F_3 \right) + \frac{3}{5} \left(G_0 F_3 + G_3 F_0 \right)^2 + \left(G_0 G_3 - F_0 F_3 \right)^2 \right] \quad (C18)$$

$$F3 = \frac{2}{3} \int dr R r^2 \left[\left(G_0^2 - F_0^2 \right) \left(G_3^2 - F_3^2 \right) + \frac{4}{5} \left(G_0 F_0 G_3 F_3 \right) + \frac{4}{5} \left(G_0 F_3 + G_3 F_0 \right)^2 \right] \quad (C19)$$

$$F4 = \frac{2}{3} \int dr R r^2 \left[\left(G_0^2 - F_0^2 \right) \left(G_3^2 - F_3^2 \right) - \frac{4}{15} \left(G_0 F_0 G_3 F_3 \right) - \frac{2}{3} \left(G_0 G_3 - F_0 F_3 \right)^2 + \frac{1}{5} \left(G_0 F_3 + G_3 F_0 \right)^2 \right] \quad (C20)$$

$$F5 = \int dr R r^2 \left(G_0^2 + F_0^2 \right)^2 \quad (C21)$$

$$F6 = 2 \int dr R r^2 \left[\left(G_0^2 - F_0^2 \right) \left(G_2^2 - F_2^2 \right) - \frac{4}{3} \left(G_0 F_0 G_2 F_2 \right) - \frac{1}{3} \left(G_0 G_2 - F_0 F_2 \right)^2 + \left(F_0 G_2 + G_0 F_2 \right)^2 \right] \quad (C22)$$

$$F7 = \frac{2\sqrt{2}}{3} \int dr R r^2 \left(F_0 F_3 + G_0 G_3 \right) \left(G_0 G_2 + F_0 F_2 \right) \quad (C23)$$

$$F8 = \sqrt{\frac{8}{27}} \int dr R r^2 \left[-2 \left(G_0 F_0 \right) \left(G_2 F_3 + G_3 F_2 \right) + \left(G_0 G_3 - F_0 F_3 \right) \left(G_0 G_2 - F_0 F_2 \right) \right] \quad (C24)$$

$$F9 = \frac{2}{3} \int dr R r^2 \left[\left(G_0^2 - F_0^2 \right) \left(G_3^2 - F_3^2 \right) + \frac{4}{15} \left(G_0 G_3 F_0 F_3 \right) - \frac{1}{3} \left(G_0 G_3 - F_0 F_3 \right)^2 + \frac{3}{5} \left(G_0 F_3 + G_3 F_0 \right)^2 \right] \quad (C25)$$

$$F10 = \frac{2}{3} \int dr R r^2 \left[-\frac{16}{15} \left(G_0 F_0 G_3 F_3 \right) - \frac{2}{3} \left(G_0 G_3 - F_0 F_3 \right)^2 + \frac{2}{5} \left(G_0 F_3 + G_3 F_0 \right)^2 \right] \quad (C26)$$

$$F11 = 2 \int dr R r^2 \left[\left(G_0^2 - F_0^2 \right) \left(G_2^2 - F_2^2 \right) + \frac{4}{3} \left(G_0 F_0 G_2 F_2 \right) - \frac{2}{3} \left(G_0 G_2 - F_0 F_2 \right)^2 \right] \quad (C27)$$

$$F12 = 2 \int dr R r^2 \left[-\frac{8}{3} \left(G_0 F_0 G_2 F_2 \right) + \frac{1}{3} \left(G_0 G_2 - F_0 F_2 \right)^2 + \left(F_0 G_2 + G_0 F_2 \right)^2 \right] \quad (C28)$$

$$F13 = 2 \int dr R r^2 \left[\frac{1}{3} \left(G_0 G_2 + F_0 F_2 \right)^2 - \left(G_0 F_2 - G_2 F_0 \right)^2 \right] \quad (C29)$$

$$F14 = \sqrt{\frac{8}{27}} \int dr R r^2 \left[2 \left(G_0 F_0 \right) \left(G_2 F_3 + G_3 F_2 \right) + \left(G_0 G_3 - F_0 F_3 \right) \left(G_0 G_2 - F_0 F_2 \right) \right] \quad (C30)$$

$$F15 = -\sqrt{\frac{32}{27}} \int dr R r^2 \left(G_0 F_0 \right) \left(G_2 F_3 + G_3 F_2 \right) \quad (C31)$$

The radial functions $G_0(r), F_0(r), G_2(r), F_2(r), G_3(r)$ and $F_3(r)$ are defined in Eqs. (2.15) to (2.17) and are shown in Figure 1 for $m_q = 7.5$ and 300 MeV.

APPENDIX D: FOURIER TRANSFORMED MESON-QUARK SOURCE FUNCTION

Definitions of operators $\mathcal{O}_{MN}^{a,l}(\hat{q})$ and corresponding functions $F^l(|\vec{q}|)$ appearing in the Fourier transformed meson-quark source function, Eq. (4.4), are presented in this Appendix. The functions $F^l(|\vec{q}|)$ in Eq. (4.4) are defined as

$$F_{SS}^{l=1}(|\vec{q}|) = (-i8\pi) \int dr r^2 j_1(|\vec{q}|r) \frac{g_\phi}{(g_\chi \chi(r))^2} G_0(r) F_0(r) \quad (D1)$$

$$F_{PP}^{l=1}(|\vec{q}|) = (-i8\pi) \int dr r^2 j_1(|\vec{q}|r) \frac{g_\phi}{(g_\chi \chi(r))^2} G_2(r) F_2(r) \quad (D2)$$

$$F_{AA}^{l=1}(|\vec{q}|) = (-i8\pi) \int dr r^2 j_1(|\vec{q}|r) \frac{g_\phi}{(g_\chi \chi(r))^2} G_3(r) F_3(r) \quad (D3)$$

$$F_{AP}^{l=1}(|\vec{q}|) = (-i4\pi) \int dr r^2 j_1(|\vec{q}|r) \frac{g_\phi}{(g_\chi \chi(r))^2} (G_2(r) F_3(r) + G_3(r) F_2(r)) \quad (D4)$$

$$F_{AA}^{l=3}(|\vec{q}|) = (+i8\pi) \int dr r^2 j_3(|\vec{q}|r) \frac{g_\phi}{(g_\chi \chi(r))^2} G_3(r) F_3(r), \quad (D5)$$

where $j_l(|\vec{q}|r)$ is the usual spherical Bessel function of order l . The corresponding single quark operators $\mathcal{O}_{MN}^{a,l}(\hat{q})$ are

$$\mathcal{O}_{SS}^{a,l=1}(\hat{q}) = \lambda^a \left(\chi_m^{1/2} \right)^\dagger \hat{q} \cdot \vec{\sigma} \left(\chi_m^{1/2} \right) \quad (D6)$$

$$\mathcal{O}_{PP}^{a,l=1}(\hat{q}) = \lambda^a \left(\chi_m^{1/2} \right)^\dagger \hat{q} \cdot \vec{\sigma} \left(\chi_m^{1/2} \right) \quad (D7)$$

$$\mathcal{O}_{AA}^{a,l=1}(\hat{q}) = \lambda^a \frac{1}{5} \left(\chi_m^{3/2} \right)^\dagger \sigma_b^{[3/2,1/2]} \hat{q} \cdot \vec{\sigma} \sigma_b^{[1/2,3/2]} \left(\chi_m^{3/2} \right) \quad (D8)$$

$$\mathcal{O}_{AP}^{a,l=1}(\hat{q}) = \lambda^a \left(\chi_m^{3/2} \right)^\dagger \hat{q} \cdot \vec{\sigma}^{[3/2,1/2]} \left(\chi_m^{1/2} \right) \quad (D9)$$

$$\mathcal{O}_{AA}^{a,l=3}(\hat{q}) = \lambda^a \left(\chi_m^{3/2} \right)^\dagger \left[\hat{q} \cdot \vec{\sigma}^{[3/2,1/2]} \hat{q} \cdot \vec{\sigma} \hat{q} \cdot \vec{\sigma}^{[1/2,3/2]} - \frac{1}{5} \sigma_b^{[3/2,1/2]} \hat{q} \cdot \vec{\sigma} \sigma_b^{[1/2,3/2]} \right] \left(\chi_m^{3/2} \right) \quad (D10)$$

In the above definitions χ_m^j is a $2j+1$ component spinor analogous to the two component Pauli spinors for $j=1/2$ case. The operators σ_a with $a = 1, 2, 3$ are the usual Pauli matrices, while $\sigma_a^{[1/2,3/2]}$ are (2×4) Multipole Transition Matrices given by [47],

$$\sigma_1^{[1/2,3/2]} = \frac{1}{\sqrt{6}} \begin{pmatrix} -\sqrt{3} & 0 & 1 & 0 \\ 0 & -1 & 0 & \sqrt{3} \end{pmatrix}, \quad (D11)$$

$$\sigma_2^{[1/2,3/2]} = \frac{-i}{\sqrt{6}} \begin{pmatrix} \sqrt{3} & 0 & 1 & 0 \\ 0 & 1 & 0 & \sqrt{3} \end{pmatrix}, \quad (D12)$$

$$\sigma_3^{[1/2,3/2]} = \sqrt{\frac{2}{3}} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad (D13)$$

with the property,

$$\left(\sigma_a^{[1/2,3/2]} \right)^\dagger = \sigma_a^{[3/2,1/2]}. \quad (D14)$$

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TABLE CAPTIONS

TABLE I. Eigenvalues ϵ_i of $H_{\text{Toy}}^{(0)}$ as defined in Eq (2.18) for $i = S, P$ and A quark spinor states with masses $m_q = 7.5$ and 300 MeV.

TABLE II. Values of scalar charge \mathcal{S}_i defined in Eq. (2.19) for $i = S, P$ and A with quark masses $m_q = 7.5$ MeV and 300 MeV.

TABLE III. Well-established low-lying negative parity baryon states considered in this work [39]. Spin-parity assignments and experimental mass ranges of the resonances are shown in columns two and three, respectively. In column four the $L - S$ coupled basis with the notation $|70, ^{2S+1}SU(3)_J\rangle$ allowed by the projection prescription discussed in Section III A are shown for each state.

TABLE IV. Matrix elements of $H_{\text{Toy}}^{(0)}$, Eq. (2.14), and $H_{\text{t Hooft}}$, Eq. (2.27), in MeV for negative parity baryons. These matrix elements are obtained using $m_s = m_{u,d} = 7.5$ MeV and using the projection prescription discussed in Section III A. The values of model parameters used in the calculation are given in Section II A. The fourth column shows the matrix elements of OGE interaction obtained in the MIT bag model evaluated in the SU(3) limit with vanishing current quark masses. The OGE matrix elements are multiplied by the strong coupling constant α_s .

TABLE V. Bare masses and SU(6) spin-flavor wave functions of negative parity baryons N^* , Δ^* , Λ^* and Σ^* in the CCM. Masses and wave functions obtained by using the toy model Hamiltonian, $H_{\text{Toy}}^{(0)}$, alone are shown in the second column while the entries in the third column are calculated using the bare CCM Hamiltonian, $H_{\text{Bare}} = H_{\text{toy}}^{(0)} + H_{\text{t Hooft}}$. Baryon masses in MeV are indicated within parenthesis and the three-quark wave functions are expressed in the $L - S$ coupled basis. Column four shows the bare masses of hyperons in MeV after being corrected for $m_s - m_{u,d}$ mass difference as explained in Section II B.

TABLE 6. Relative percentages of spin-flavor contents in the $L - S$ coupled basis for low-lying negative parity $J^P = 3/2^-$ and $1/2^-$ Λ^* hyperons in the CCM of this work, the MIT bag model with massless quarks and the NRQM [32].

m_q (MeV)	ϵ_S (MeV)	ϵ_P (MeV)	ϵ_A (MeV)
7.5	340	550	490
300	595	772	743

TABLE I.

m_q (MeV)	\mathcal{S}_S	\mathcal{S}_P	\mathcal{S}_A
7.5	0.61	0.33	0.54
300	0.79	0.63	0.71

TABLE II.

Resonance	J^P	Mass Range (MeV)	$ 70, {}^{2S+1} SU(3)_J\rangle$
$N(1675)$	$5/2^-$	1670 – 1685	$ {}^4 8_{5/2}\rangle$
$N(1700)$	$3/2^-$	1650 – 1750	$ {}^4 8_{3/2}\rangle$ $ {}^2 8_{3/2}\rangle$
$N(1520)$	$3/2^-$	1515 – 1530	$ {}^4 8_{3/2}\rangle$ $ {}^2 8_{3/2}\rangle$
$N(1650)$	$1/2^-$	1640 – 1680	$ {}^4 8_{1/2}\rangle$ $ {}^2 8_{1/2}\rangle$
$N(1535)$	$1/2^-$	1520 – 1555	$ {}^4 8_{1/2}\rangle$ $ {}^2 8_{1/2}\rangle$
$\Delta(1700)$	$3/2^-$	1670 – 1770	$ {}^2 10_{3/2}\rangle$
$\Delta(1620)$	$1/2^-$	1615 – 1675	$ {}^2 10_{1/2}\rangle$
$\Lambda(1830)$	$5/2^-$	1810 – 1830	$ {}^4 8_{5/2}\rangle$
$\Lambda(1690)$	$3/2^-$	1685 – 1695	$ {}^4 8_{3/2}\rangle$ $ {}^2 8_{3/2}\rangle$ $ {}^2 1_{3/2}\rangle$
$\Lambda(1520)$	$3/2^-$	1518 – 1520	$ {}^4 8_{3/2}\rangle$ $ {}^2 8_{3/2}\rangle$ $ {}^2 1_{3/2}\rangle$
$\Lambda(1800)$	$1/2^-$	1720 – 1850	$ {}^4 8_{1/2}\rangle$ $ {}^2 8_{1/2}\rangle$ $ {}^2 1_{1/2}\rangle$
$\Lambda(1670)$	$1/2^-$	1660 – 1680	$ {}^4 8_{1/2}\rangle$ $ {}^2 8_{1/2}\rangle$ $ {}^2 1_{1/2}\rangle$
$\Lambda(1405)$	$1/2^-$	1400 – 1410	$ {}^4 8_{1/2}\rangle$ $ {}^2 8_{1/2}\rangle$ $ {}^2 1_{1/2}\rangle$
$\Sigma(1775)$	$5/2^-$	1770 – 1780	$ {}^4 8_{5/2}\rangle$
$\Sigma(1940)$	$3/2^-$	1900 – 1950	$ {}^4 8_{3/2}\rangle$ $ {}^2 8_{3/2}\rangle$ $ {}^2 10_{3/2}\rangle$
$\Sigma(1670)$	$3/2^-$	1665 – 1685	$ {}^4 8_{3/2}\rangle$ $ {}^2 8_{3/2}\rangle$ $ {}^2 10_{3/2}\rangle$
$\Sigma(1750)$	$1/2^-$	1730 – 1800	$ {}^4 8_{1/2}\rangle$ $ {}^2 8_{1/2}\rangle$ $ {}^2 10_{1/2}\rangle$

TABLE III.

State	$H_{\text{Toy}}^{(0)}$	$H_{\text{t Hooft}}$	H_{OGE}
$N(3/2^-)$	$\begin{pmatrix} 1503 & 21 \\ 21 & 1483 \end{pmatrix}$	$\begin{pmatrix} -31 & 0.2 \\ 0.2 & -31 \end{pmatrix}$	$\alpha_s \begin{pmatrix} 60 & -1 \\ -1 & 47 \end{pmatrix}$
$N(1/2^-)$	$\begin{pmatrix} 1523 & 13 \\ 13 & 1503 \end{pmatrix}$	$\begin{pmatrix} 178 & 89 \\ 89 & -12 \end{pmatrix}$	$\alpha_s \begin{pmatrix} -23 & -15 \\ -15 & -24 \end{pmatrix}$
$\Lambda(3/2^-)$	$\begin{pmatrix} 1470 & 0 & 0 \\ 0 & 1503 & 21 \\ 0 & 21 & 1483 \end{pmatrix}$	$\begin{pmatrix} -90 & 0 & 0 \\ 0 & -31 & 0.2 \\ 0 & 0.2 & -31 \end{pmatrix}$	$\alpha_s \begin{pmatrix} -12 & 0 & 0 \\ 0 & 60 & -1 \\ 0 & -1 & 47 \end{pmatrix}$
$\Lambda(1/2^-)$	$\begin{pmatrix} 1530 & 0 & 0 \\ 0 & 1523 & 13 \\ 0 & 13 & 1503 \end{pmatrix}$	$\begin{pmatrix} -10 & 0 & 0 \\ 0 & 178 & 89 \\ 0 & 89 & -12 \end{pmatrix}$	$\alpha_s \begin{pmatrix} -61 & 0 & 0 \\ 0 & -23 & -15 \\ 0 & -15 & -24 \end{pmatrix}$
$\Sigma(3/2^-)$	$\begin{pmatrix} 1497 & 0 & 0 \\ 0 & 1503 & 21 \\ 0 & 21 & 1483 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & -31 & 0.2 \\ 0 & 0.2 & -31 \end{pmatrix}$	$\alpha_s \begin{pmatrix} 85 & 0 & 0 \\ 0 & 60 & -1 \\ 0 & -1 & 47 \end{pmatrix}$
$\Sigma(1/2^-)$	$\begin{pmatrix} 1477 & 0 & 0 \\ 0 & 1523 & 13 \\ 0 & 13 & 1503 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 178 & 89 \\ 0 & 89 & -12 \end{pmatrix}$	$\alpha_s \begin{pmatrix} 55 & 0 & 0 \\ 0 & -23 & -15 \\ 0 & -15 & -24 \end{pmatrix}$

TABLE IV.

State	$H_{\text{Toy}}^{(0)}$	$H_{\text{Bare}} = H_{\text{Toy}}^{(0)} + H_{\text{t Hooft}}$	Δs
$N(5/2^-)$	$ (1470)\rangle = ^4 8_{5/2}\rangle$	$ (1470)\rangle = ^4 8_{5/2}\rangle$	N/A
$N(3/2^-)_1$	$ (1518)\rangle = 0.823 ^4 8_{3/2}\rangle + 0.567 ^2 8_{3/2}\rangle$	$ (1485)\rangle = 0.846 ^4 8_{3/2}\rangle + 0.534 ^2 8_{3/2}\rangle$	N/A
$N(3/2^-)_2$	$ (1473)\rangle = -0.567 ^4 8_{3/2}\rangle + 0.823 ^2 8_{3/2}\rangle$	$ (1439)\rangle = -0.534 ^4 8_{3/2}\rangle + 0.846 ^2 8_{3/2}\rangle$	N/A
$N(1/2^-)_1$	$ (1529)\rangle = 0.897 ^4 8_{1/2}\rangle + 0.442 ^2 8_{1/2}\rangle$	$ (1742)\rangle = 0.927 ^4 8_{1/2}\rangle + 0.376 ^2 8_{1/2}\rangle$	N/A
$N(1/2^-)_2$	$ (1496)\rangle = -0.442 ^4 8_{1/2}\rangle + 0.897 ^2 8_{1/2}\rangle$	$ (1450)\rangle = -0.376 ^4 8_{1/2}\rangle + 0.927 ^2 8_{1/2}\rangle$	N/A
$\Delta(3/2^-)$	$ (1497)\rangle = ^2 10_{3/2}\rangle$	$ (1497)\rangle = ^2 10_{3/2}\rangle$	N/A
$\Delta(1/2^-)$	$ (1477)\rangle = ^2 10_{1/2}\rangle$	$ (1477)\rangle = ^2 10_{1/2}\rangle$	N/A
$\Lambda(5/2^-)$	$ (1470)\rangle = ^4 8_{5/2}\rangle$	$ (1470)\rangle = ^4 8_{5/2}\rangle$	1654
$\Lambda(3/2^-)_1$	$ (1518)\rangle = 0.823 ^4 8_{3/2}\rangle + 0.567 ^2 8_{3/2}\rangle$	$ (1485)\rangle = 0.846 ^4 8_{3/2}\rangle + 0.534 ^2 8_{3/2}\rangle$	1667
$\Lambda(3/2^-)_2$	$ (1473)\rangle = -0.567 ^4 8_{3/2}\rangle + 0.823 ^2 8_{3/2}\rangle$	$ (1439)\rangle = -0.534 ^4 8_{3/2}\rangle + 0.846 ^2 8_{3/2}\rangle$	1618
$\Lambda(3/2^-)_3$	$ (1470)\rangle = ^2 1_{3/2}\rangle$	$ (1380)\rangle = ^2 1_{3/2}\rangle$	1557
$\Lambda(1/2^-)_1$	$ (1529)\rangle = 0.897 ^4 8_{1/2}\rangle + 0.442 ^2 8_{1/2}\rangle$	$ (1742)\rangle = 0.927 ^4 8_{1/2}\rangle + 0.376 ^2 8_{1/2}\rangle$	1925
$\Lambda(1/2^-)_2$	$ (1530)\rangle = ^2 1_{1/2}\rangle$	$ (1520)\rangle = ^2 1_{1/2}\rangle$	1676
$\Lambda(1/2^-)_3$	$ (1496)\rangle = -0.442 ^4 8_{1/2}\rangle + 0.897 ^2 8_{1/2}\rangle$	$ (1450)\rangle = -0.376 ^4 8_{1/2}\rangle + 0.927 ^2 8_{1/2}\rangle$	1628
$\Sigma(5/2^-)$	$ (1470)\rangle = ^4 8_{5/2}\rangle$	$ (1470)\rangle = ^4 8_{5/2}\rangle$	1640
$\Sigma(3/2^-)_1$	$ (1497)\rangle = ^4 10_{3/2}\rangle$	$ (1497)\rangle = ^4 10_{3/2}\rangle$	1665
$\Sigma(3/2^-)_2$	$ (1518)\rangle = 0.823 ^4 8_{3/2}\rangle + 0.567 ^2 8_{3/2}\rangle$	$ (1485)\rangle = 0.846 ^4 8_{3/2}\rangle + 0.534 ^2 8_{3/2}\rangle$	1638
$\Sigma(3/2^-)_3$	$ (1473)\rangle = -0.567 ^4 8_{3/2}\rangle + 0.823 ^2 8_{3/2}\rangle$	$ (1439)\rangle = -0.534 ^4 8_{3/2}\rangle + 0.846 ^2 8_{3/2}\rangle$	1601
$\Sigma(1/2^-)_1$	$ (1529)\rangle = 0.897 ^4 8_{1/2}\rangle + 0.442 ^2 8_{1/2}\rangle$	$ (1742)\rangle = 0.927 ^4 8_{1/2}\rangle + 0.376 ^2 8_{1/2}\rangle$	1881
$\Sigma(1/2^-)_2$	$ (1477)\rangle = ^2 10_{1/2}\rangle$	$ (1477)\rangle = ^2 10_{1/2}\rangle$	1652
$\Sigma(1/2^-)_3$	$ (1496)\rangle = -0.442 ^4 8_{1/2}\rangle + 0.897 ^2 8_{1/2}\rangle$	$ (1450)\rangle = -0.376 ^4 8_{1/2}\rangle + 0.927 ^2 8_{1/2}\rangle$	1619

TABLE V.

Model (\rightarrow)	Chiral Confining Model			
Hyperon state (\downarrow)	Bare Mass (MeV)	$ ^2 1_J\rangle$ (%)	$ ^4 8_J\rangle$ (%)	$ ^2 8_J\rangle$ (%)
$\Lambda(3/2^-)_1$	1667	0.0	71.6	28.4
$\Lambda(3/2^-)_2$	1618	0.0	28.4	71.6
$\Lambda(3/2^-)_3$	1557	100	0.0	0.0
$\Lambda(1/2^-)_1$	1925	0.0	85.9	14.1
$\Lambda(1/2^-)_2$	1676	100	0.0	0.0
$\Lambda(1/2^-)_3$	1628	0.0	14.1	85.9
Model (\rightarrow)	MIT Bag Model			
Hyperon state (\downarrow)	Mass (MeV)	$ ^2 1_J\rangle$ (%)	$ ^4 8_J\rangle$ (%)	$ ^2 8_J\rangle$ (%)
$\Lambda(3/2^-)_1$	1705	0.0	80.8	19.2
$\Lambda(3/2^-)_2$	1620	0.0	19.2	80.8
$\Lambda(3/2^-)_3$	1526	100	0.0	0.0
$\Lambda(1/2^-)_1$	1598	0.0	100	0.0
$\Lambda(1/2^-)_2$	1563	0.0	0.0	100
$\Lambda(1/2^-)_3$	1552	100	0.0	0.0
Model (\rightarrow)	Non-Relativistic Quark Model			
Hyperon state (\downarrow)	Physical Mass (MeV)	$ ^2 1_J\rangle$ (%)	$ ^4 8_J\rangle$ (%)	$ ^2 8_J\rangle$ (%)
$\Lambda(3/2^-)_1$	1880	0.2	98.0	1.2
$\Lambda(3/2^-)_2$	1690	16.0	1.4	82.8
$\Lambda(3/2^-)_3$	1490	82.8	1.0	16.0
$\Lambda(1/2^-)_1$	1800	3.2	72.3	25.0
$\Lambda(1/2^-)_2$	1650	10.0	33.6	56.3
$\Lambda(1/2^-)_3$	1490	81.0	0.4	18.5

TABLE VI.

FIGURE CAPTIONS

FIG 1. The upper, $G(r)$, and the lower, $F(r)$, radial functions for quark spinors in states $S_{1/2}$, $P_{1/2}$ and $P_{3/2}$ as defined in Eqs. (2.15) to (2.17). The solid and dashed lines indicate radial functions obtained with quark masses of $m_q = 7.5$ MeV and $m_q = 300$ MeV, respectively. a) $S_{1/2}$ radial functions $G_0(r)$ and $F_0(r)$. The radial profile of the $\chi^{(0)}$ field is also shown for comparison using the dot-dashed line. b) $P_{1/2}$ radial functions $G_2(r)$ and $F_2(r)$. c) $P_{3/2}$ radial functions $G_3(r)$ and $F_3(r)$. Note that the distance shown in the horizontal axis is given in mesic units of $1/\mu$.

FIG 2. Normalized $NN\pi$ dipole form factor $R(NN\pi) \equiv G_{NN\pi}^{l=1}(|\vec{q}|)/G_{NN\pi}^{l=1}(0)$ for a) $N = N(5/2^-)$, b) $N = N(3/2^-)_1$ (dashed line), $N = N(3/2^-)_2$ (solid line) and c) $N = N(1/2^-)_1$ (dashed line), $N = N(1/2^-)_2$ (solid line).

FIG 3. Normalized $\Lambda_1 Nk$ dipole form factor $R(\Lambda_1 Nk) \equiv G_{\Lambda_1 Nk}^{l=1}(|\vec{q}|)/G_{\Lambda_1 Nk}^{l=1}(0)$ for a) $N = N(3/2^-)_1$ (dashed line), $N = N(3/2^-)_2$ (solid line) and b) $N = N(1/2^-)_1$ and $N = N(1/2^-)_2$ (solid line). Note that $R(\Lambda_1 Nk)$ involving $N(1/2^-)$ states are identical.





